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Phenomenal Understanding on Drop Impact in Accidental Conditions

Kuniharu Kinoshita, Nuclear Fuel Transport Co.
Kenichi Hakozaki. Nuclear Fuel Transport Co.

ABSTRACT

IAEA regulations require nine-meter vertical, horizontal, corner, and oblique drop tests. As regards the oblique drop test, it is difficult and costly to evaluate inclination angle dependence on package damage by means of a real drop test or dynamic FEM analysis, such as LS-DYNA.

A simple mechanical model is developed for quantitative understanding of oblique drop phenomena. This mechanical model is based on the equation of motion for spring and dumper, which describes shock absorbing and rebounding actions. In order to estimate the most severe inclination angle for an oblique drop, this mechanical model is linked to velocity data evaluated by LS-DYNA. This mechanical model is able to reproduce LS-DYNA results.

Consequently, it is concluded that the mechanical model is useful for reproducing oblique drop phenomena.

INTRODUCTION

In regard to the oblique drop test, IAEA material states:

> "Experience suggests that the effect of secondary impact is often more severe for slender and rigid packages, including:
>
> (a) A package with an aspect ratio (length to diameter) larger than 5, but sometimes even as low as 2;…."

Unfortunately, the explanation does not provide clarity for foreseeing what kind of packages will be subject to more severe impact or how to estimate severe drop conditions. Especially, estimation of the inclination angle conditions of an oblique drop is problem. If we define the most severe inclination angle conditions by dynamic analysis, we must perform many analyses with different inclination angles. Such consideration is generally very costly.

Therefore, a simple mechanical model to represent oblique drop phenomena is developed.

This mechanical model is based on the equation of motion for spring and dumper, which describes shock absorbing and rebounding behavior. We focus on velocity data evaluated by LS-DYNA or obtained in a real drop test such as rebound velocity, which can be defined by rebound height.

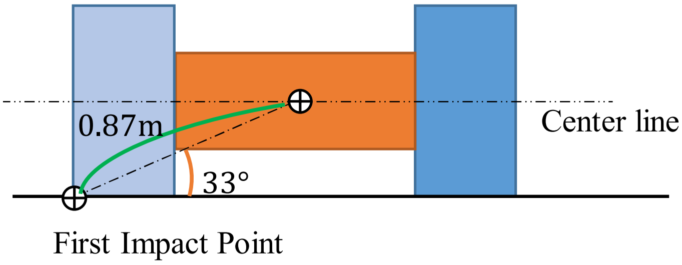
This mechanical model can reproduce the LS-DYNA results, such as velocity of center of gravity and angular velocity. Using little velocity data (only 3 cases of LS-DYNA), we have been successful in finding the most severe inclination angle conditions.

RESULTS OF DYNAMIC ANALYSIS

In order to find the inclination angle dependence for an oblique drop test, we performed the following dynamic analyses using LS-DYNA.

<u>Overview of Analysis Model</u> (dummy package):

- Cylindrical figure, Length: 2.0m, Diameter: 1.0m
- Weight: 4400kg, Moment of inertia: 1200kg · m^2
- Length between the center of gravity and first impact point: 0.87m
- Angle of center line to line between the center of gravity and first impact point: 33°
  The angle is defined in the drawing at right.



<u>Initial Conditions</u>:

- Drop height: 9.3m (Velocity at first impact: 13.6m/s)
- Initial inclination angles: 3 cases (5°, 10°, and 15°)

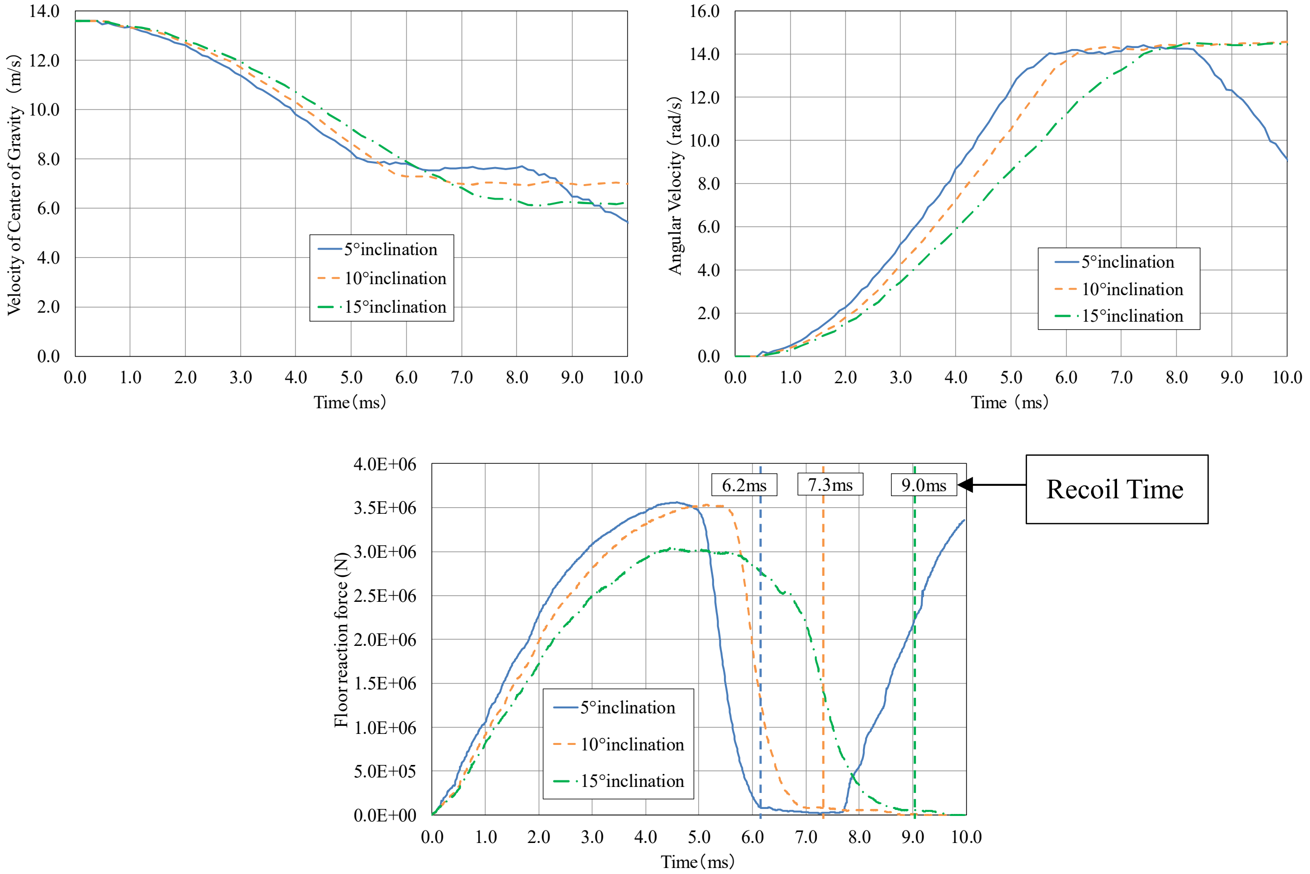


Figure 1. Results of Dynamic Analysis (LS-DYNA)

Figure 1 shows the velocity of the center of gravity (CG) and the angular velocity around CG, which are calculated using the velocity at two observation points along the center line of package and the floor reaction force. Based on Figure 1, it is concluded that:

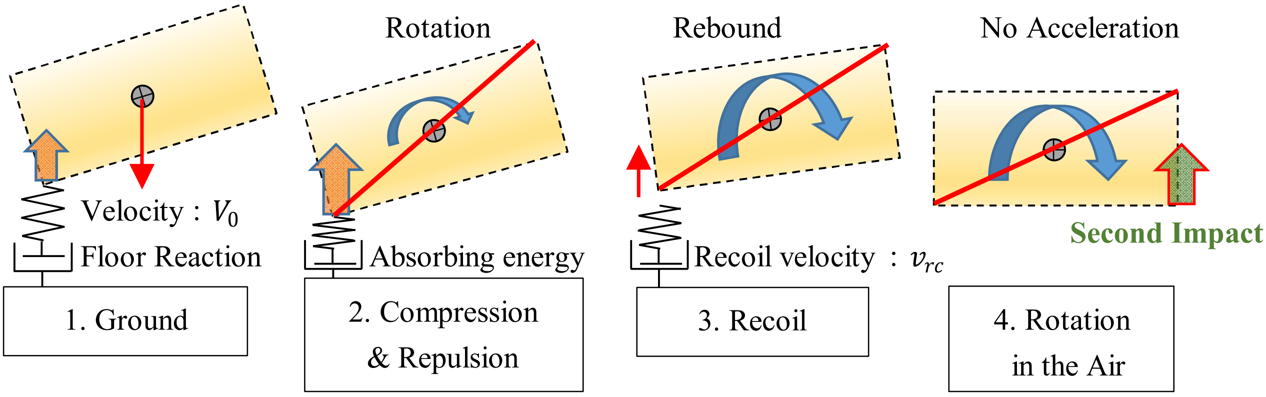
- Rebound action occurred when the floor reaction force was zero (recoil time: t_{rc}).
- The larger the inclination angle is, the smaller CG velocity at recoil.

- The angular velocity is almost same against to the inclination angle.
- The larger inclination angle is, the longer the duration of the floor reaction force.
- After recoil, angular velocity and CG velocity do not change. Therefore, it seems that the package rotates in the air after recoil.

It is considered that "slap down" in the oblique drop test occurs when the velocity of the second impact point (opposite end of package) becomes faster than the initial drop velocity, because the velocity at the second impact point is calculated by combining the circumferential velocity and CG velocity.

MECHANICAL MODEL OF OBLIQUE DROP

Following the moving behavior as shown in the results of the dynamic analysis, the following mechanical model is considered as shown in Figure 2.



Note: Diagonal red line represents the package.

Figure 2. Mechanical Model Concept

The mechanical model assumes the following:

- When the first impact point touches the ground, time starts to count: $t = 0$ (ms)
- Initial velocity at ground is the free drop velocity of 9.3m: $V_0 = 13.6$ (m/s)
- There are both spring and dumper at the first impact point. The dumper represents the shock absorbing behavior due to deformation of impact area of the package.
- The spring and dumper behavior are described using the equation of motion for spring and dumper.
- Just after the first impact point touches ground, the spring is compressed, and rotation around CG is started.
- After full compression, the spring is repulsed.
- Impact point of package recoils from ground floor at recoil time (t_{rc}) with recoil velocity (v_{rc}).
- At recoil time, floor reaction force (or acceleration) is zero. ($\frac{d^2x}{dt^2}(t_{rc}) = 0$)
- After recoil, the package rotates in the air, and the angular velocity does not change because there is no force to accelerate the rotation.

Floor Reaction Force Model Based on Spring and Dumper

When a package collides with the floor surface, the package decelerates due to the floor reaction force, then it rebounds upward. This means that the package behaves like a spring.

Because the upward velocity of the rebound is less than the initial drop velocity (13.6m/s), the package also behaves as a dumper.

Therefore, the floor reaction force can be modeled using the spring and dumper. It is noted that, in this mechanical model, the gravitational acceleration is ignored because deceleration generated by the floor reaction is more than several hundred times greater than the gravitational acceleration.

The equation of motion for spring and dumper is as follows.

$$m \cdot \frac{d^2x}{dt^2} + R \cdot \frac{dx}{dt} + k \cdot x = 0 \qquad \text{(Equation 1)}$$

Where; m: Mass, R: Coefficient of dumper, k : Spring constant, x: Virtual displacement

The solutions (Equation 1) under initial conditions of $\frac{dx}{dt}(0) = V_0$, $\frac{d^2x}{dt^2} = 0$ are as follows.

$$v(t) = \frac{dx}{dt} = K \cdot \frac{V_0}{\lambda} \cdot exp(-\gamma \cdot t) \cdot cos(\lambda \cdot t - \emptyset) \qquad \text{(Equation 2)}$$

$$\alpha(t) = \frac{dv}{dt} = \frac{d^2x}{dt^2} = -K^2 \cdot \frac{V_0}{\lambda} \cdot exp(-\gamma \cdot t) \cdot sin(\lambda \cdot t) \qquad \text{(Equation 3)}$$

Where; $R = 2 \cdot m \cdot \gamma$, $k = m \cdot K^2$, $\lambda^2 = K^2 - \gamma^2 > 0$, $sin(\emptyset) = \frac{\gamma}{K}$, $cos(\emptyset) = \frac{\lambda}{K}$

Mechanical Model for Slap Down Behavior of Package

A rigid rod having the same mass and moment of inertia as the package is considered.

As shown in Figure 3, the initial velocity of the inclined rod is 13.6m/s (V_0), and the inclined rod does not rotate before collision.

There is no friction from the floor. In Figure 3, θ_0 is the initial inclination angle of oblique drop.

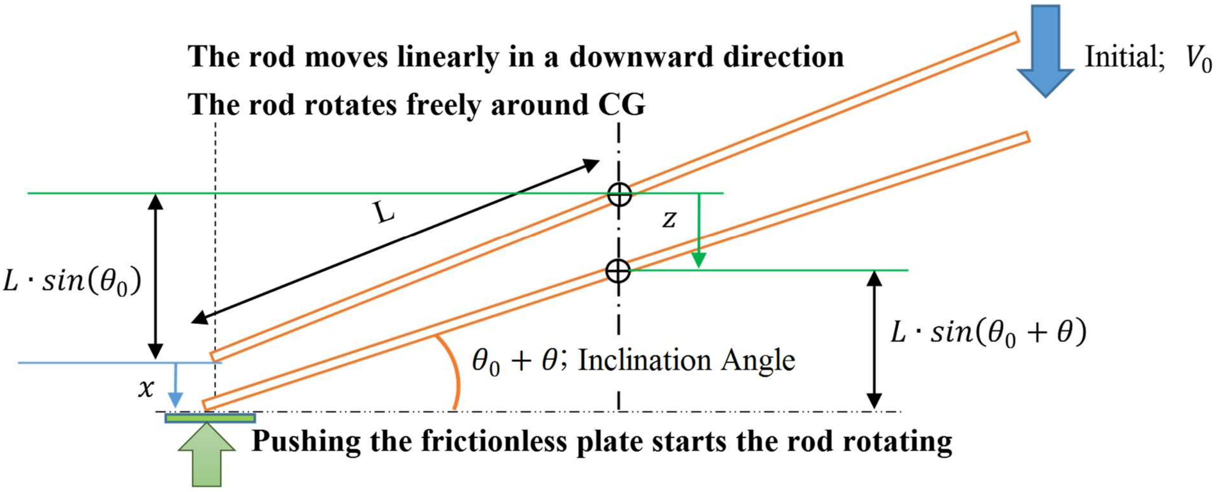


Figure 3. Mechanical Model for Slap Down Behavior

Based on the geometrical shape, taking into account a sufficiently small amount of angle change(θ), the relation between x and z is defined as follows.

$$x + L \cdot sin(\theta_0) = z + L \cdot sin(\theta_0 + \theta)$$

$$\therefore z = x - L \cdot cos(\theta_0) \cdot \theta \quad (\because \quad cos(\theta) \approx 1 \text{ and } sin(\theta) \approx \theta \quad)$$

The following equations are derived by differentiating the above equation.

$$\frac{dz}{dt} = \frac{dx}{dt} - L \cdot cos(\theta_0) \cdot \frac{d\theta}{dt}$$

$$\frac{d^2z}{dt^2} = \frac{d^2x}{dt^2} - L \cdot cos(\theta_0) \cdot \frac{d^2\theta}{dt^2} \qquad \text{(Equation 4)}$$

Force acting on the package is considered to be only the floor reaction force because the gravitational acceleration is negligible in comparison to floor reaction. Therefore, CG of the package receives the same force as the floor reaction force.

$$m_{pc} \cdot \frac{d^2z}{dt^2} = -F_{fl} \qquad \text{(Equation 5)}$$

Where; m_{pc}: Mass of package, $\frac{d^2z}{dt^2}$: Acceleration of CG, F_{fl}: Floor reaction force

Based on acceleration of the first impact point, equivalent mass (m_{eq}) is defined as follows.

$$m_{eq} \cdot \frac{d^2x}{dt^2} = -F_{fl} \qquad \text{(Equation 6)}$$

Where; $\frac{d^2x}{dt^2}$: Acceleration of first impact point

Based on the balance of moment around center of gravity, following equation is derived.

$$I_{pc} \cdot \frac{d^2\theta}{dt^2} = L \cdot cos(\theta_0) \cdot \left(-F_{fl}\right) \qquad \text{(Equation 7)}$$

Where; I_{pc}: Moment of inertia of the package, $\frac{d^2\theta}{dt^2}$: Angular acceleration

It is noted that during the first impact $L \cdot cos(\theta_0)$ is assumed to be constant because the duration of the first impact is very short (on the order of milliseconds).

Consolidating Equations 4 through 7, the following equations are derived.

$$\frac{dz}{dt} = \frac{m_{eq}}{m_{pc}} \cdot \left(\frac{dx}{dt} - V_0\right) + V_0 \quad \text{: Velocity of CG of the package}$$

$$\frac{d\theta}{dt} = \frac{1}{L \cdot cos(\theta_0)} \cdot \left(1 - \frac{m_{eq}}{m_{pc}}\right)\left(\frac{dx}{dt} - V_0\right) \quad \text{: Angular velocity around CG of the package}$$

$$m_{eq} = \frac{I_{pc} \cdot m_{pc}}{m_{pc} \cdot \left(L \cdot cos(\theta_0)\right)^2 + I_{pc}} \quad \text{: Equivalent mass at first impact point}$$

REPRODUCTION OF DYNAMIC ANALYSIS RESULTS BY THE MECHANICAL MODEL

CG velocity, angular velocity and floor reaction acceleration are reproduced by the aforementioned mechanical model. An operation performed to define parameters γ and K (or λ) is referred to as Fitting .

Method of Fitting

In reproducing slap down behavior, the focus is on the following results obtained from LS-DYNA.

- Recoil time ($t = t_{rc}$): Defined as the result of floor reaction that becomes zero.
- Recoil velocity: Defined as the result of vertical velocity of the first impact point at t_{rc}.

Coefficients of λ and γ, which are defined in Equations 2 and 3, can be derived using recoil time and recoil velocity as follows.

$$\frac{dx^2}{dt^2}(t_{rc}) = -K^2 \cdot \frac{V_0}{\lambda} \cdot exp(-\gamma \cdot t_{rc}) \cdot sin(\lambda \cdot t_{rc}) = 0$$

$$\therefore \lambda = \pi / t_{\mathrm{rc}}$$

$$\frac{dx}{dt}(t_{rc}) = K \cdot \frac{V_0}{\lambda} \cdot exp(-\gamma \cdot t_{rc}) \cdot cos(\lambda \cdot t_{rc} - \emptyset)$$

$$\therefore \gamma = -\frac{1}{t_{rc}} \cdot log\left(\frac{v_L(t_{rc})}{-V_0}\right)$$

The Fitting parameters are shown in Table 1.

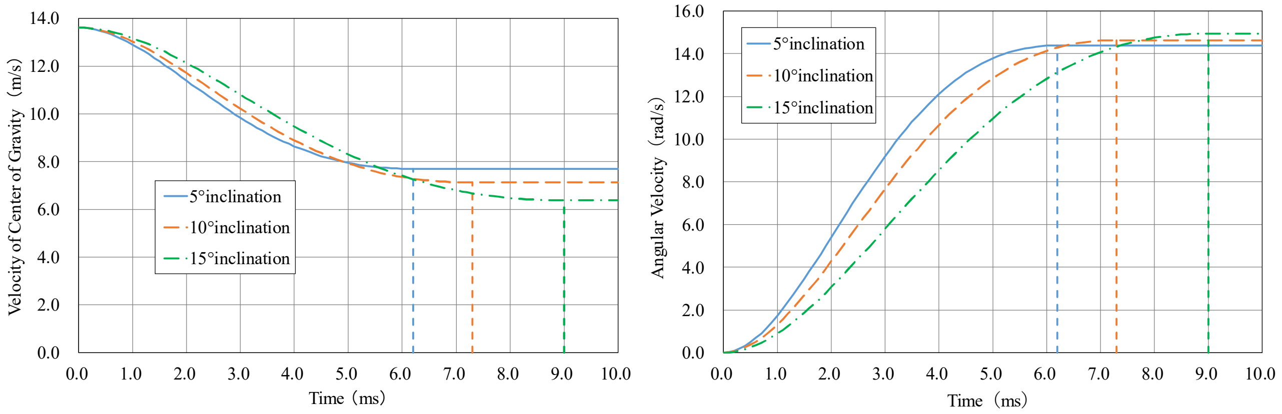
Table 1. Fitting parameters

Inclination angle	5°	10°	15°
Recoil Time	6.2 ms	7.3 ms	9.0 ms
Recoil velocity at observation point 1)	2.9	3.5	4.2
Recoil velocity at first impact point 2)	2.13	2.15	2.28
γ	225	174	111
K	555	487	366

1) Observation points with LS-DYNA are set along the centre line of the package.
2) Because the first impact point disappears due to deformation, the recoil velocity at the first impact point is corrected using the distance and angle from the observation points with LS-DYNA.

Results of Fitting

CG velocity and angular velocity calculated by the mechanical model are shown in Figure 4.



(a) CG velocity by Mechanical Model (b) Angular velocity by Mechanical Model

Figure 4. Results of Fitting

From Figure 4, the following results are found.

- The larger the inclination angle is, the smaller the CG velocity at recoil.
- The angular velocity is almost the same and does not depend on the inclination angle.

Therefore, it is concluded that the mechanical model is able to reproduce the tendency well.

Discussion

There is a small discrepancy between the LS-DYNA results and the mechanical model results. The cause of the discrepancy is considered to be due to the following reasons.

- In this model, R value (coefficient of dumper) is assumed to be constant during the first impact. However, because the real package deforms, R value may change over time.
- In this model, the inclination angle is assumed to be constant during the first impact. However, in real behavior, the balance of moment may change due to a change in the inclination angle.

Additionally, rebound velocities are used as input data for this Fitting, but there is another way where the angular velocity defined by LS-DYNA may also be used as input data for Fitting.

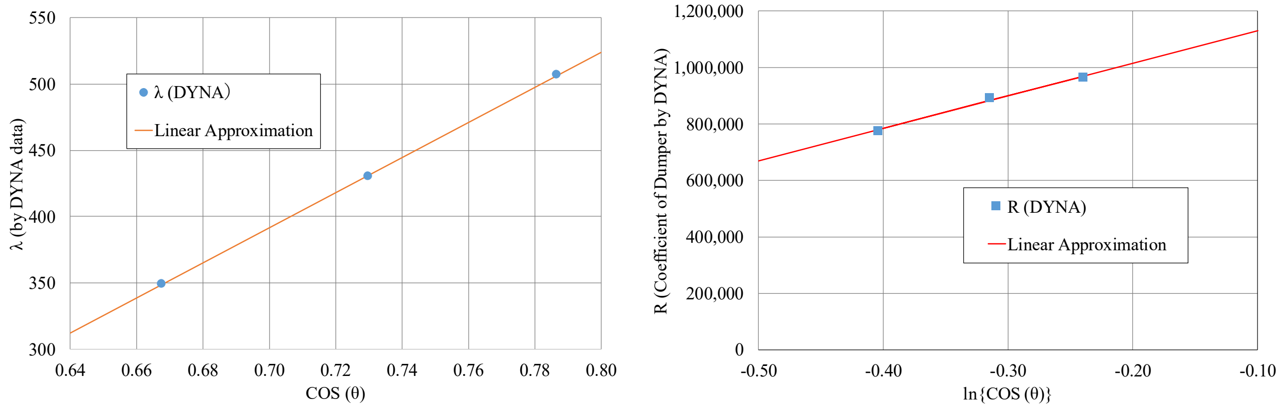
ESTIMATION OF THE MOST SEVERE INCLINATION ANGLE OF OBLIQUE DROP

The velocity of the second impact point changes due to the initial inclination angle in an oblique drop. This mechanical model can estimate the velocity of the second impact point using two values γ and K. So, if the inclination angle dependency of the γ and K values are found, the most severe inclination angle of oblique drop can be defined by the mechanical model.

By using the of LS-DYNA results, the following relations may be found as shown in Figure 5.

- λ value is proportional to $cos(\theta_0)$.
- R value is proportional to $ln(cos(\theta_0))$.

It has been remarked that the aforementioned relations may not be general ones and the relations may depend on packaging design. Because the R value signifies the energy absorbing characteristic which is defined not only by the material at the first impact point but also its structure.

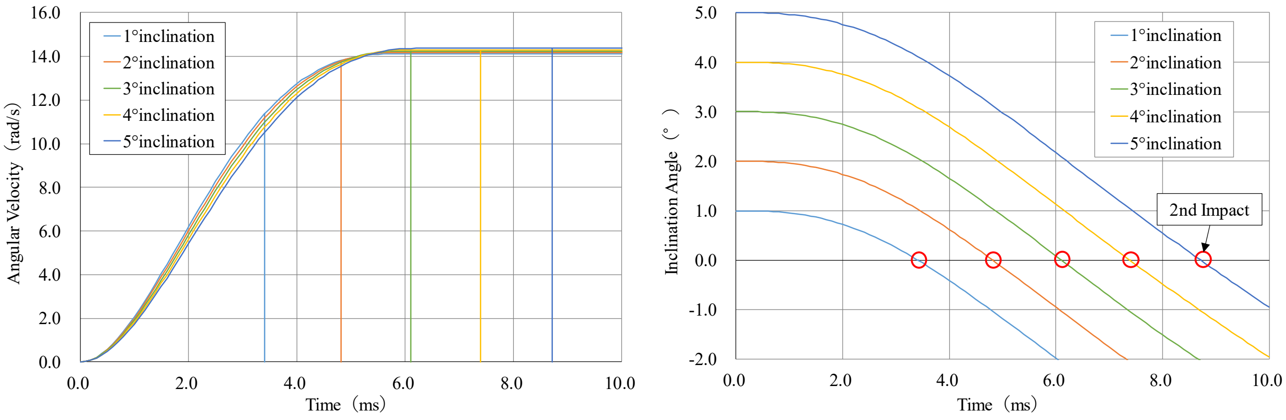


(a) Relation between λ and $cos(\theta_0)$ (b) Relation between $R(= 2m_{eq}\gamma)$ and $\ln(cos(\theta_0))$

Figure 5. Inclination angle dependency for parameters of the mechanical model

Estimation of Second Impact Velocity

Based on the aforementioned relationship between initial inclination angle and parameters (λ and R), angular velocities are calculated for several initial inclination angles. Then, the change in the inclination angles over time is calculated as shown in Figure 6.



(a) Estimated angular velocity (b) Estimated angle change

Figure 6. Estimation of package rotation

If the second impact point grounds when the inclination angle is zero, the second impact time can be defined by the inclination angle using the aforementioned method (Figure 6) and the maximum velocity of the second impact point can be estimated from the circumferential velocity and the CG velocity.

In the case of a small initial inclination angle, the second impact may occur before the angular velocity reaches the maximum as shown in Figure 6.

Taking into account the result of inclination angle change, the vertical velocity of the second impact point is calculated as shown in Figure 7. In Figure 7, the red circle indicates the vertical velocity value at the second impact time.

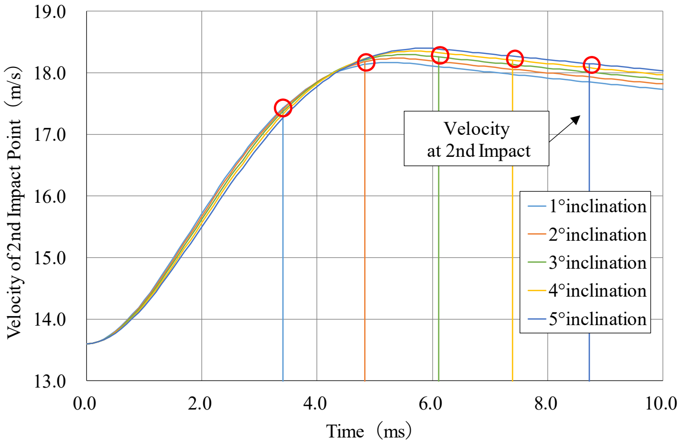


Figure 7. Vertical velocity of second impact point in case of small initial inclination angle

The most severe condition of oblique drop is considered to be the initial inclination angle where the velocity of the second impact point is at the maximum

The velocity at the second impact time with several different initial inclination angles are evaluated by the aforementioned method. The most severe initial inclination angle is estimated as shown in Figure 8.

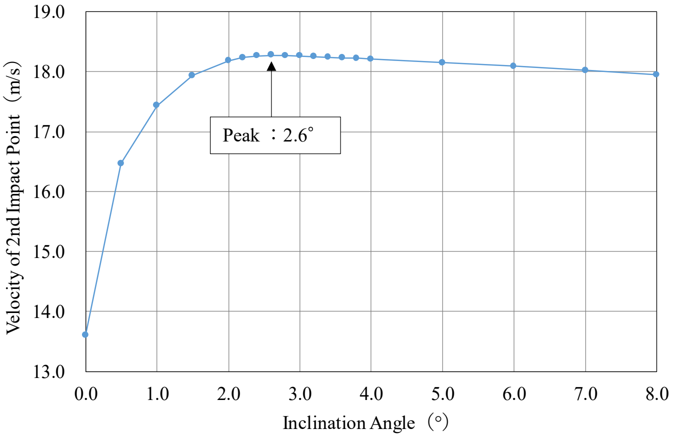


Figure 8. Estimation of the most severe initial inclination angle

Discussion

In a case of the aforementioned estimation, it is assumed inclination angle of package is zero at the second impact time (t_{2nd}) as shown in Figure 6.

However, in the case of real drop test, the first impact point may be deformed and the configuration of the package at the second impact is as shown in Figure 9. For example, in the case of 5° oblique drop conditions from LS-DYNA, $\theta(t_{2nd})$ is approximately 2.7°.

Taking into account the configuration in Figure 9, the velocity at the second impact time is smaller, and the most severe initial inclination angle becomes greater than the maximum value estimated in Figure 8.

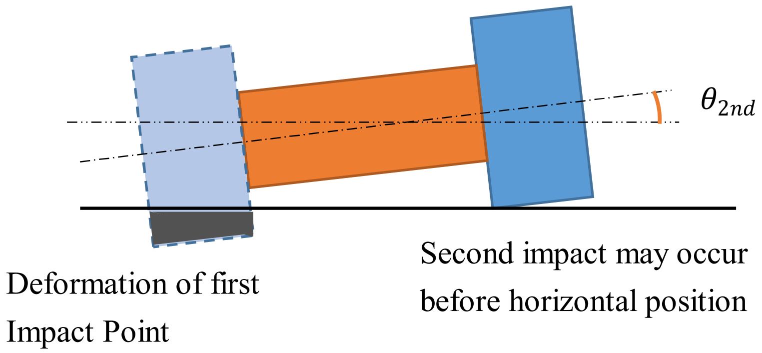


Figure 9. Schematic configuration showing actual second impact condition

CONCLUSIONS

With regard to the oblique drop test, it is difficult and costly to evaluate the inclination angle dependence on package damage using real drop tests or dynamic FEM analysis, such as LS-DYNA.
In order to make simple estimation of the most severe initial inclination angle of oblique drop, a mechanical model is developed. The mechanical model is based on the equation of motion for spring and dumper, which describes shock absorbing and rebounding actions.
Then, based on the result from LS-DYNA, several input data for the mechanical model, such as recoil time and recoil velocity, are prepared. This model can reproduce the LS-DYNA results.

Consequently, it is concluded that the mechanical model can reproduce the slap dawn phenomena.
Finally, based on the LS-DYNA results of three cases, this mechanical model was inferred to successfully estimate the most severe conditions. These conditions are defined by the initial inclination angle of oblique drop, and the conditions are the initial inclination angle where the velocity of second impact point is at the maximum.
It is concluded that the mechanical model is useful for reproducing oblique drop phenomena.

References

1. IAEA, Advisory Material for the IAEA Regulations for the Safe Transport of Radioactive Material.
2. T. Quercetti, V. Ballheimer, G. Wieser, “Analytical, Numerical and Experimental Investigations on the Impact Behaviour of Packagings for the Transport of Radioactive Material under Slap Down” PATRAM2001, Chicago, Sep.3-7 2001.