Perturbation Analysis for Demonstration of Reactivity in Criticality Safety Analyses

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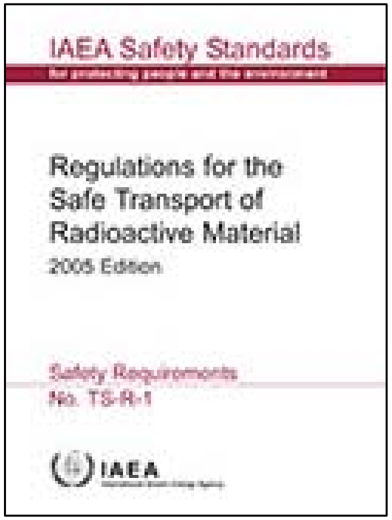
Criticality Safety Analysis Guidance

- TS-R-1, paragraph 673

  Where the chemical or physical form, isotopic composition, mass or concentration, moderation ratio or density, or geometric configuration is not known, the assessments of paras 677–682 [refer to the assessment of an isolated, individual package and package arrays for normal and accident conditions] *shall be performed assuming that each parameter that is not known has the value which gives the* <u>*maximum neutron multiplication*</u> *consistent with the known conditions and parameters in these assessments.*



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Overview

- Perturbation Methods
  - Direct
  - Analytical
- Applications
  - Contents parameter selection
    - Burnable absorber rod distribution
  - Validation evaluations
  - Uncertainty analysis
    - Allowances

Perturbation Theory Benefit

- Evaluate the relative worth of a parameter
- Determine the sensitivity of the k_{eff} with respect to changes in the system
- Results in simplified contents specification
  - Minimize potential, unnecessary restrictions that transport package requirements would impose on the fuel bundle design
  - Simplify package approval document

Direct Perturbation Method

- Unknown parameter
  - varied and evaluated through complete calculation sets repeated

- Worth of variation determined by comparison

$$\rho_{worth} = \frac{k_{\text{nominal}} - k_{perturbation}}{k_{\text{nominal}}}$$

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Analytical Perturbation Method

- Use of an analytical tool
  - SCALE sensitivity and uncertainty (S/U) analysis tool TSUNAMI-3D
  - Calculates adjoint-based first-order linear perturbation theory sensitivity coefficients
    - Sensitivity Coefficient $$\frac{\Delta keff \,/\, keff}{\Delta\Sigma \,/\, \Sigma}$$
  - Validated by central difference direct perturbation

Applications

- Criticality safety transport analyses
  - Contents parameter selection
    - Burnable absorber rod distribution
  - Validation evaluations
  - Uncertainty analysis
    - Allowances

Contents Parameter: BWR Burnable Absorber (BA) Rods

- Background
  - Burnable absorber rods within a BWR assembly are utilized to achieve desired core performance objectives
- Methodology

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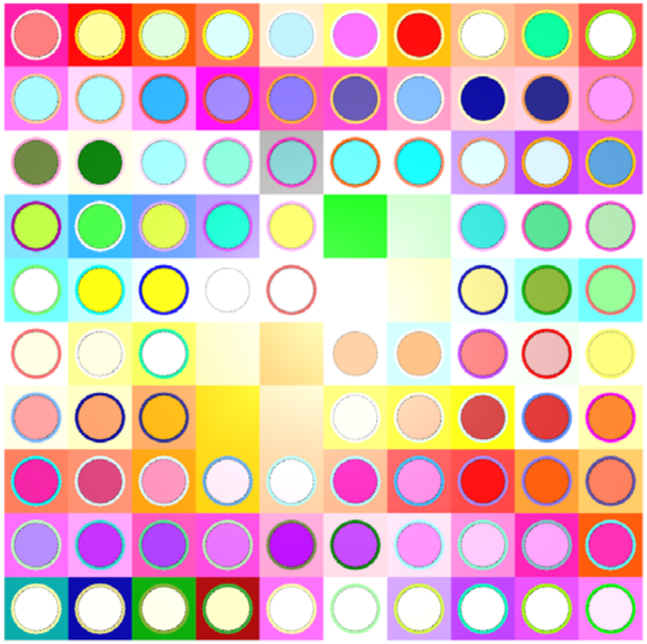
Burnable Absorber Rod Evaluation

- SCALE Models
  - Each fuel rod 5.0 wt% enriched UO_2 + 0.1 wt% Gd_2O_3
    - Minimizes flux alteration
  - Each fuel rod material has a unique material identifier
    - Accounts for individual location effects
- Sensitivity coefficients integrated over energy and region for ^{157}Gd



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BA Rod Pattern Selection Example

- Identify least worth locations
- Symmetry rule
  - Calculate average worth among pairs
- Select pattern for criticality safety analyses

Validation of Analytic Perturbation

Central difference direct perturbation

$$S_{k,\alpha} = \frac{k_{\alpha^{+x\%}} - k_{\alpha^{-x\%}}}{k_{nominal}} \times \frac{100(\%)}{2x(\%)}$$

Case	Number Density Multiplier	Sensitivity Coefficient	Percent Difference
Fuel Rod	1 – nominal	-0.190 ± 0.0003	1.4%
	±10%	-0.192 ± 0.0004	
Infinite Bundle Array	1 – nominal	-0.115 ± 0.0013	10.7%
	±10%	-0.128 ± 0.0012	

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Uncertainty Allowance Assessment

- Two allowance categories

(1) Analytical perturbation uncertainty
  - Material and fabrication tolerances

(2) Direct perturbation uncertainty
  - Geometric or material representation

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Analytical Perturbation: Material and Fabrication Tolerances

- Reaction rate

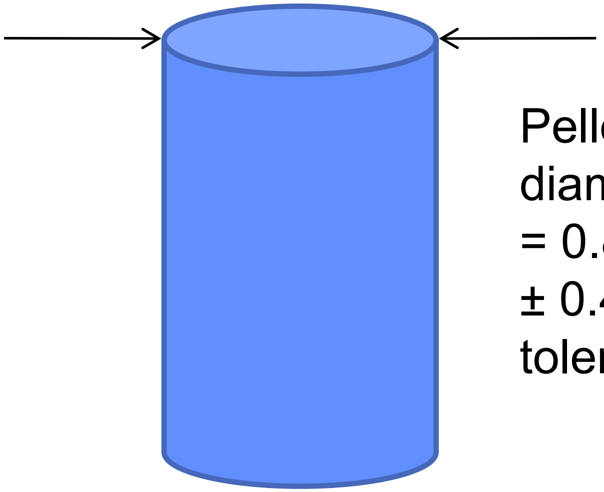
$$R = \phi\Sigma = \phi N\sigma \qquad \Delta R = \phi \frac{N_A}{M} \Delta\rho\sigma = \phi N \Delta\sigma$$

- Sensitivity coefficient

$$\frac{\Delta keff / keff}{\Delta\Sigma / \Sigma} = \frac{\Delta keff / keff}{\Delta\rho / \rho}$$

- Tolerance correlation

$$-\frac{\Delta V}{V} \equiv \frac{\Delta\rho}{\rho}$$



Pellet diameter = 0.85 cm ± 0.4% tolerance

Analytical Perturbation: Total Material and Fabrication Uncertainty

- Uncertainty associated with region

$$\left(\frac{\Delta keff}{keff}\right)_i = \left[\frac{\Delta keff / keff}{\Delta\Sigma/\Sigma}\right]_i \cdot \left(\frac{\Delta V}{V}\right)_i$$

- Simple summation

$$\left(\frac{\Delta keff}{keff}\right)_{TOTAL} = \sum_i \left(\frac{\Delta keff}{keff}\right)_i$$

- Analytical perturbation uncertainty total

$$\Delta k_{u,analytical} = \left(\frac{\Delta keff}{keff}\right)_{TOTAL} \times k_p \quad \text{where} \quad k_p = 1.0$$

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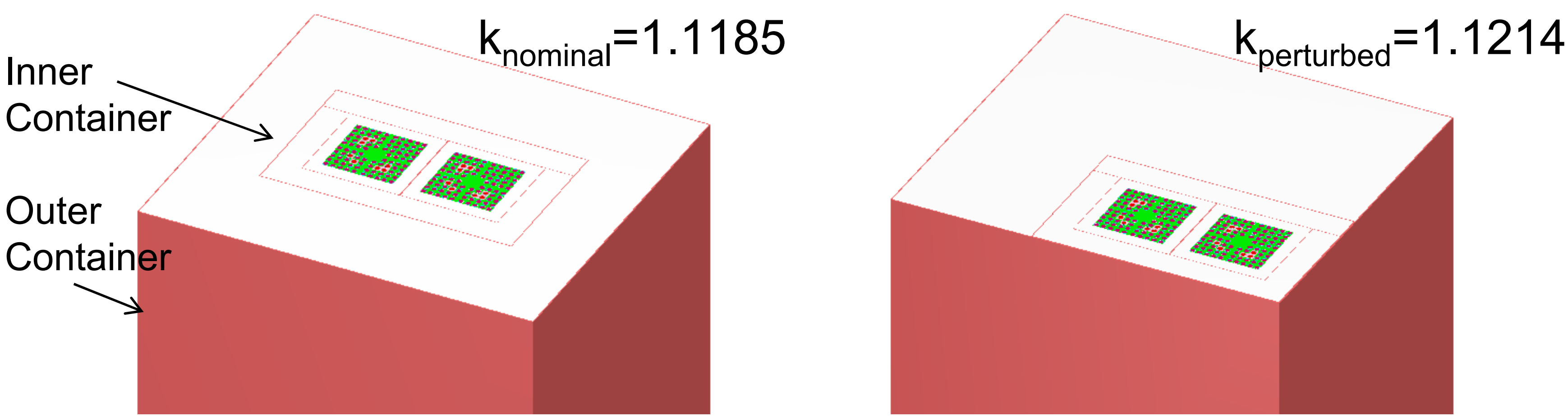
Direct Perturbation: Geometric or Material Representations

- Direct difference between the nominal and perturbed case

$$\Delta k_{u,direct} = k_{perturbed} - k_{nominal}$$

- Example geometric representation: $\Delta k_{u,direct} = 0.0029$



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Total Uncertainty

- Total combined uncertainty

  $\Delta k_u = \Delta k_{u,analytical} + \Delta k_{u,direct}$

  (1) Analytical perturbation uncertainty

  $\Delta k_{u,analytical}$

  - Material and fabrication tolerances

  (2) Direct perturbation uncertainty

  $\Delta k_{u,direct}$

  - Geometric or material representation

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Conclusions

Expanding the application of perturbation theory

- More efficiently define maximum k_{eff}
- Simplify contents specification

- Use analytical perturbation method for evaluating BA rod position worth in a BWR lattice
  - Valid method of optimizing the contents parameters
- Development of the uncertainty methodology shows relations between k_{eff} to nuclide density

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