Estimation of Package Temperatures During Hypothetical Accident Thermal Test Conditions*

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INTRODUCTION

Knowledge of package maximum temperatures during normal transport and hypothetical accident thermal test conditions is required to ensure that a package for the transportation of radioactive materials is designed adequately to meet the requirements specified in the Code of Federal Regulations, Title 10, Part 71 (10 CFR §71). The testing procedure for the hypothetical accident thermal event, specified in 10 CFR §71.73, is to expose the whole package to a radiation environment at 800°C (1475°F) for a period of 30 mins, with the ambient temperature before and after the 30-min test held constant at 38°C (100°F).

To determine the temperatures of a package during the hypothetical accident thermal event, the designers and reviewers perform detailed analyses with computer codes. Because detailed modeling of the package and solving the thermal diffusion equation with computer codes require considerable time and effort, simple conservative estimating procedures are therefore preferred because the very detailed and time-consuming numerical computation can be avoided if the results of a simple conservative estimation can show that a package meets the requirements.

The simple procedure presented in this paper provides quick and conservative estimation of package maximum temperatures, $T_{max}(x)$, at a distance x from the surface, during hypothetical accident thermal test conditions. The analytical solutions for temperature in a semi-infinite solid and in an infinitely long circular cylinder, subjected to a step surface-temperature boundary condition, are applied to estimate the maximum temperature. The procedure is applicable to all packages, with or without an internal heat source, that have rectangular or cylindrical thermal insulating overpacks. Examples, mathematical proof, and qualitative arguments are presented to show that the proposed method is conservative and easy to use and therefore can be used for estimating package maximum temperatures without performing a very lengthy and time-consuming computer analysis.

*This work was supported by the U.S. Department of Energy, Office of Facility Safety Analysis, under Contract W-31-109-Eng-38.

PROCEDURE

The estimating procedure uses three steps: (1) calculate the geometrical and physical parameters, r/R R^2/α, and/or x^2/α for the location of interest in the package; (2) read the maximum temperature, $T_{max\text{-}analytic,q=0}$, from Figures 1 and 2 presented in this paper; and (3) add the heat source contribution from the initial steady-state temperature distribution, i.e.,

$$T_{max\text{-}estimation} = T_{max\text{-}analytic,\ q=0} + (\ T_{package\text{-}steady} \ - T_0\ ). \tag{1}$$

Here, r is the radius at the location of interest; R is the radius of the cylindrical surface of the package; x and α are the width and thermal diffusivity of the region between the surface and the location of interest, respectively; T_0 (=100°F) is the ambient air temperature before and after the 30 min hypothetical accident thermal event; $T_{max\text{-}analytic,\ q=0,}$ derived in the Appendix and presented in Figures 1 and 2, is the maximum temperature in a semi-infinite solid or in an infinitely long circular cylinder with no internal heat source, with uniform thermal diffusivity, and a step surface temperature boundary condition of 800°C (1475°F) for a 30-min test period; and [$T_{package\text{-}steady}$ (x) $- T_0$] is the steady-state temperature rise (heat source contribution) in the package due to an internal heat source.

EXAMPLE

To illustrate the simplicity of the estimating procedure, the Mound 9859 Tritium Trap package, shown in Figure 3, is used as an example. Although this package has no internal heat source, i.e., $T_{max-estimation} = T_{max\text{-}analytic,\ q=0}$, the qualitative arguments and mathematical analysis, presented later, will show that the estimating procedure is also applicable to a package with an internal heat source.

Two values of thermal diffusivities, 3.77×10^{-4} and 1.17×10^{-3} (in^2/s), for the overpack insulation are used to show that the method is applicable over a full range of parameter values. The geometrical and physical parameters, (x^2/α), (R^2/α), and r^*(=r/R), for locations A and B shown in Figure 4, are calculated first. Here, α is the thermal diffusivity, R is the radius of the cylindrical surface, and r^* is the dimensionless radial distance. The estimated maximum temperatures $T_{max-estimation}$ at these locations for the hypothetical accident thermal test condition are then read from Figures 1 and 2.

Table 1. Comparisons of Temperatures

Location	Thermal Diffusivity α (in^2/s)	x^2/α or R^2/α (s)	$T_{max\text{-}estimation}$ from Fig. 1 or 2 (°F)	T_{max} from Numerical Simulation (°F)
A (x=3.1")	3.77×10^{-4}	2.55×10^4	188	170
	1.17×10^{-3}	8.21×10^3	355	339
B (r=2.41") r/R=0.36	3.77×10^{-4}	1.18×10^5	186	158
	1.17×10^{-3}	3.80×10^4	362	289

The results of the present method are compared with those from a very detailed numerical simulation. The computer model of the package for the numerical simulation with the HEATING module (version 6.0) of the SCALE 3.0 computer code is shown in Figure 4. The temperatures presented in Table 1 show that the present procedure provides conservative estimates.

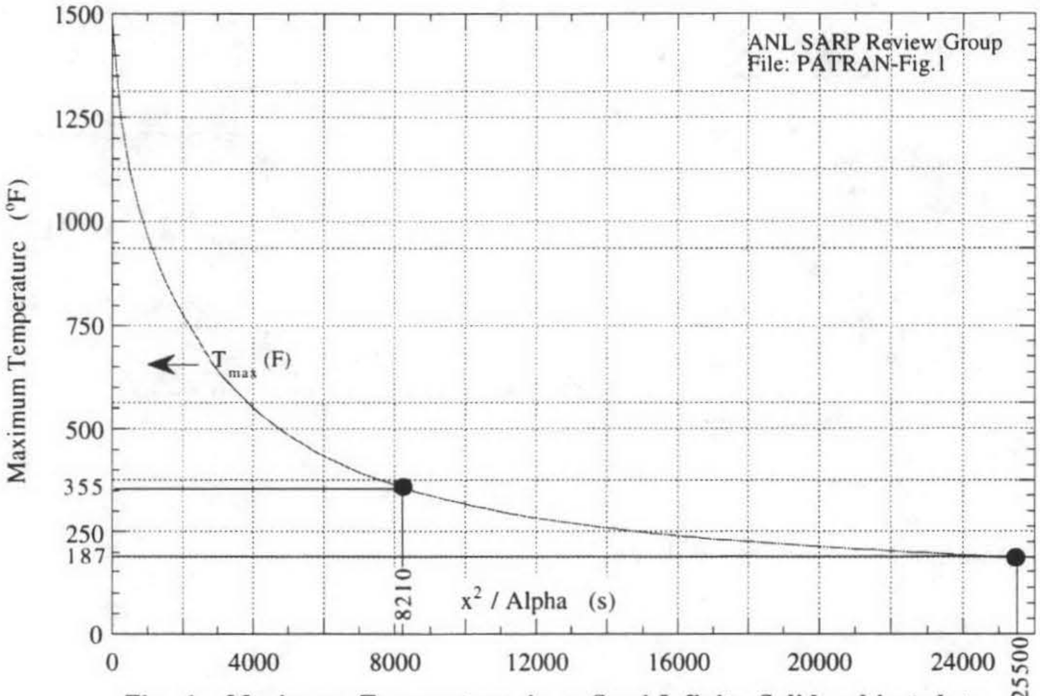

Fig. 1 Maximum Temperature in a Semi-Infinite Solid subjected to 1475 (°F) Step Temperature at the Surface for 30 minutes

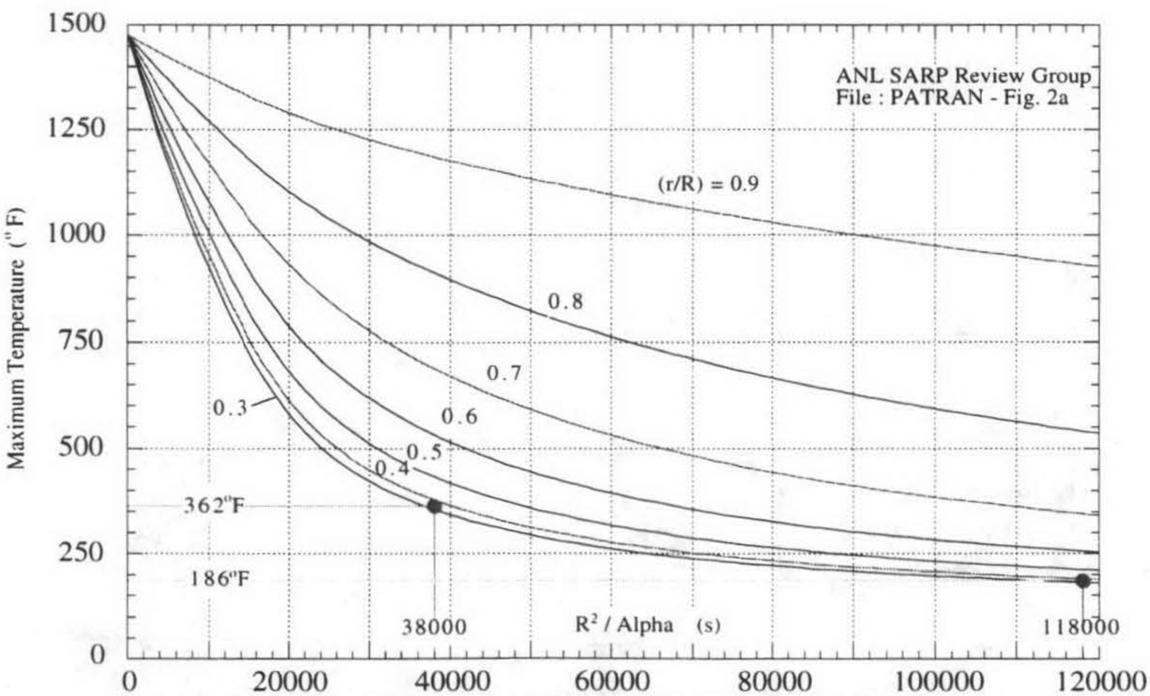

Fig. 2. Maximum Temperature in a Infinite Cylinder subjected 1475 (°F) Step Temperature at the Surface for 30 minutes.

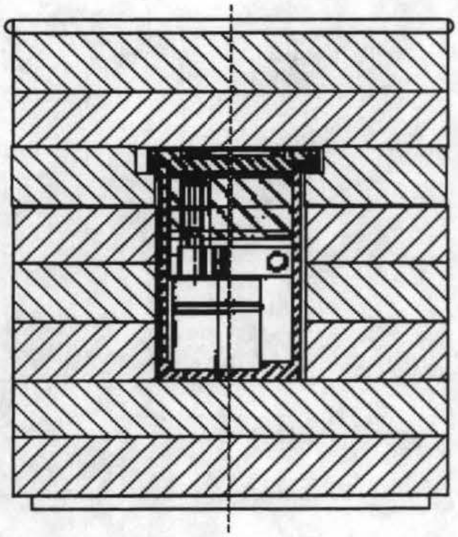

Fig. 3. Mound 9859 Tritium Package

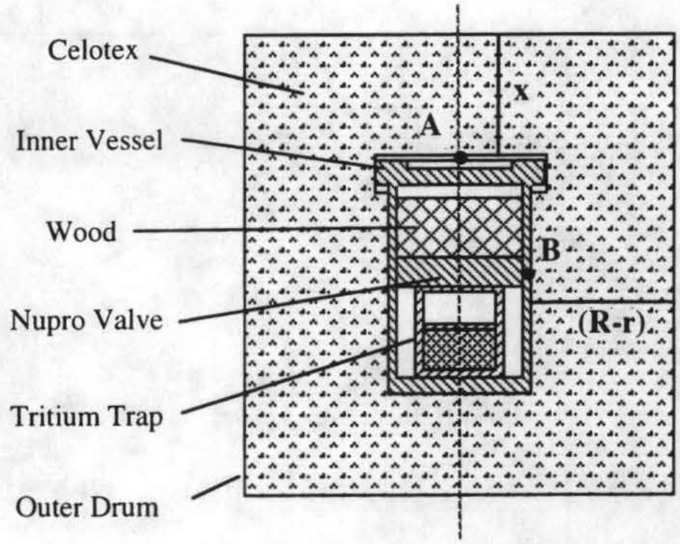

Fig. 4. Computer Model of Mound 9859 Package

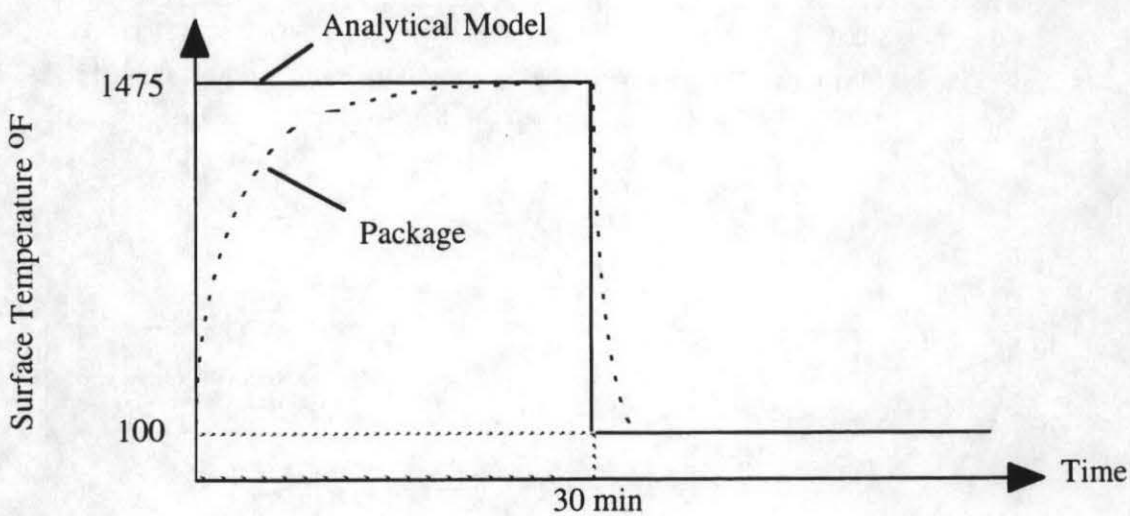

Fig. 5. Surface Temperature during Hypothetical Accident

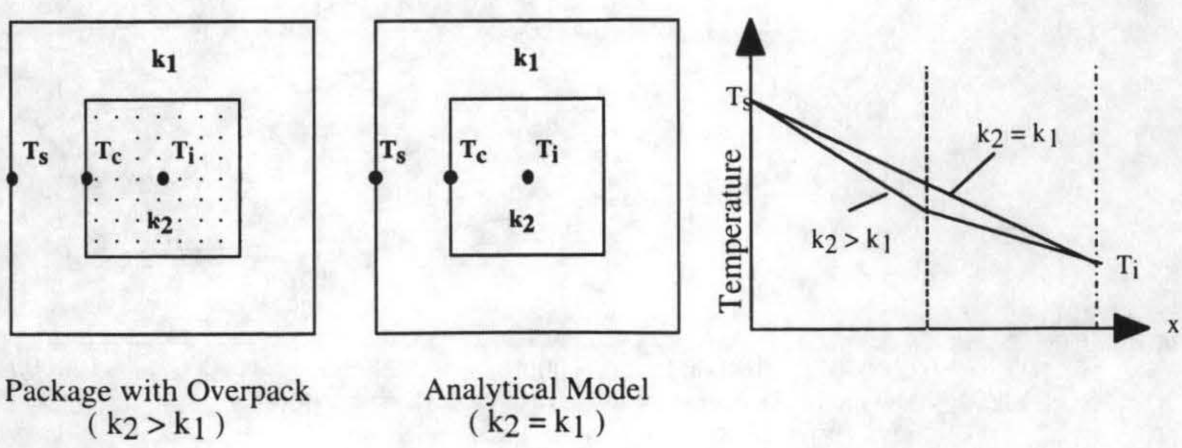

Fig. 6. Comparison of Steady-State Temperature Distributions

CONSERVATIVE PROCEDURE

Having shown that the procedure is simple and provides conservative estimate for packages with no heat source, we further justify why the analytical contribution, $T_{max\text{-}analytic,\, q=0}(x)$ in the estimating procedure will be higher than the temperature experienced by a package without an internal heat source during a hypothetical accident thermal event.

1. The surface temperature of a package under hypothetical test conditions, as specified in the Code of Federal Regulations and shown in Figure 5, will be lower than the boundary condition temperature of 800°C (1475°F) imposed in the model for $T_{max\text{-}analytic,\, q=0}$. Therefore, heat flow and temperature rise in the analytical model will be higher than in a package under the hypothetical test conditions specified in 10 CFR §71.73.

2. Simplified one-dimensional steady-state temperature distributions in two packages, one with uniform thermal diffusivity and another with two different thermal diffusivities, presented in Figure 6, show that the temperature in a package with uniform thermal diffusivity will be higher than that in a package where the outer section has lower thermal diffusivity. Because the thermal diffusivity of an overpack is always lower than that of the inner section of a package, the temperature rise in the model with uniform thermal diffusivity will be higher than that in a package under hypothetical test conditions.

Therefore,

$$T_{max\text{-}analytic,\, q=0}(x) \geq T_{max\text{-}package,\, q=0}(x)\,. \tag{2}$$

Next we show why the heat source contribution $[T_{package\text{-}steady}(x) - T_0]$ in the estimating procedure is the maximum asymptotic temperature rise from a heat source in a package. Suppose the ambient temperature of two identical packages, one with a constant internal heat source and the other with no heat source, is raised from initial temperature T_0 (assume 100°F) to T_{s1} (assume 1475°F) and then kept constant at T_{s1} for an infinite time period; then the final asymptotic temperature distributions in these packages, with the thermal diffusion equation being linear, will be

$$T_{max\text{-}asymptotic,\, q\neq 0}(x) = T_{s1} + [\, T_{package\text{-}steady}(x) - T_0\,], \text{ and} \tag{3}$$

$$T_{max\text{-}asymptotic,\, q=0}(x) = T_{s1}, \tag{4}$$

respectively. Therefore,

$$[T_{package\text{-}steady}(x) - T_0] = [T_{max\text{-}package,\, q\neq 0}(x) - T_{max\text{-}package,\, q=0}(x)]_{asymptotic}. \tag{5}$$

Furthermore, the analytical solutions presented in the Appendix show that the heat source contribution, $[\, T_{package\text{-}steady}(x) - T_0\,]$, is greater than the temperature rise due to an internal heat source that will occur in a cylinder subjected to step surface temperature boundary conditions, i.e.,

$$[\, T_{package\text{-}steady}(x) - T_0\,] \geq [\, T_{analytic,\, q\neq 0}(x,t) - T_{analytic,\, q=0}(x,t)\,]. \tag{6}$$

From Eq. 6, and knowing from Eq. 5 that the contribution $[T_{package\text{-}steady}(x) - T_0]$ is the maximum asymptotic temperature rise from a heat source, we can conclude that

$$[\, T_{package\text{-}steady}(x) - T_0\,] \geq [\, T_{package,\, q\neq 0}(x) - T_{package,\, q=0}(x)\,]. \tag{7}$$

Because

$$T_{max\text{-}analytic,\, q=0}(x) \geq T_{max\text{-}package,\, q=0}(x)\,, \text{ and} \tag{8}$$

$$[\, T_{package\text{-}steady}(x) - T_0\,] \geq [\, T_{package,\, q\neq 0}(x) - T_{package,\, q=0}(x)\,], \tag{9}$$

we can conclude that

$$T_{\text{max-estimation}}(x) = T_{\text{max-analytic, q=0}}(x) + [\, T_{\text{package-steady}}(x) - T_0 \,]$$
$$\geq T_{\text{package}}(x). \qquad (10)$$

CONCLUSIONS

The estimating procedure presented here is simple, easy to use, and provides a conservative prediction of the maximum temperature of an inner container under a hypothetical accident thermal event. The procedure is applicable to all packages, with or without an internal heat source, that have rectangular or cylindrical thermal insulating overpacks. It will be beneficial for the designers and reviewers of the transportation packages to use this estimating procedure before commencing a very lengthy and detailed computer simulation.

APPENDIX: ANALYTICAL SOLUTIONS

Cylinder With No Internal Heat Source ($q^{\cdot'''} = 0$)

The one-dimensional analytical solution for temperatures in a semi-infinite solid, initially at temperature T_0 (= 100°F), subjected to step surface temperatures T_{s1} (= 1475°F) for time t_1 (= 1800 sec), and T_{s2} (= 100°F) for time greater than t_1 (= 1800 sec), is:

$$\frac{(T - T_0)}{(T_{s1} - T_0)} = \operatorname{erfc}\left(\frac{\sqrt{x^2/\alpha}}{2\sqrt{t}}\right) \;.\;.\quad \text{for } t \leq t_1 (= 1800 \text{ sec}), \text{ and} \qquad (A.1a)$$

$$\frac{(T - T_0)}{(T_{s1} - T_0)} = \operatorname{erfc}\left(\frac{\sqrt{x^2/\alpha}}{2\sqrt{t}}\right) + \frac{(T_{s2} - T_{s1})}{(T_{s1} - T_0)} \operatorname{erfc}\left(\frac{\sqrt{x^2/\alpha}}{2\sqrt{t - t_1}}\right), \qquad (A.1b)$$

for $t \geq t_1$. Here, erfc(x) is the complementary error function. Similarly, the one-dimensional analytical solution for temperature in an infinite circular cylinder, subjected to the same boundary condition, is:

$$\frac{(T - T_0)}{(T_{s1} - T_0)} = 1 - 2\sum_{n=1}^{\infty} \psi_n \;.\;.\quad \text{for } t \leq t_1 \text{ (=1800 sec), and} \qquad (A.2a)$$

$$\frac{(T - T_0)}{(T_{s1} - T_0)} = \left(1 - 2\sum_{n=1}^{\infty} \psi_n\right) + \frac{(T_{s2} - T_{s1})}{(T_{s1} - T_0)}\left(1 - 2\sum_{n=1}^{\infty} \psi_n^{'}\right) \;.\;.\quad \text{for } t > t_1. \qquad (A.2b)$$

where

$$\psi_n = e^{\frac{-\beta_n^2 \alpha t}{R^2}} \frac{J_0(r^* \beta_n)}{\beta_n J_1(\beta_n)}, \qquad \psi_n^{'} = e^{\frac{-\beta_n^2 \alpha (t - t_1)}{R^2}} \frac{J_0(r^* \beta_n)}{\beta_n J_1(\beta_n)}.$$

$J_n(\beta)$ is the Bessel function of the first kind of order n, and $\pm (\beta_n/R)$, n = 1, 2, . . , are the roots of the Bessel function $J_0(\beta) = 0$. Maximum temperatures will occur when the temperature gradient $\partial T/\partial t = 0$, i.e., when

$$t^{1.5} \exp\left(\frac{-x^2/\alpha}{4t}\right) = (t - t_1)^{1.5} \exp\left(\frac{-x^2/\alpha}{4\,[t - t_1]}\right) \quad \text{for a semi-infinite solid,} \qquad (A.3)$$

and

$$\sum_{n=1}^{\infty} \frac{\beta_n^2 \alpha}{R^2} \psi_n' = \sum_{n=1}^{\infty} \frac{\beta_n^2 \alpha}{R^2} \psi_n \quad \text{for an infinite circular cylinder.} \tag{A.4}$$

Equations A.3 and A.4 have been solved iteratively to obtain the time t_{Tmax} when the temperature will be maximum. The maximum temperature, $T_{max\text{-}analytic,\, q=0}(x)$, obtained from Eqs. A.1 and A.2 by substituting $t = t_{Tmax}$, are presented in Figures 1 and 2.

Cylinder with Internal Heat Source ($q^{\cdot'''} \neq 0$)

Because the thermal diffusion equation is linear, the analytical solution for temperature in a cylinder with uniform internal heat source can be obtained by dividing the problem into three separate problems and summing the solution, i.e.,

$$T_{analytic,\, q\neq 0} = T_1 + T_2 + T_3 \tag{A.5}$$

The initial temperature distribution in a cylinder with a constant heat source, $q^{\cdot'''}$, is:

$$T = T_0 + (T_{si} - T_0) + \frac{q^{\cdot'''} R^2}{4K}(1 - r^{*2})$$

Here, T_0 is the ambient temperature, $(T_{si} - T_0)$ is the surface temperature rise due to heat source, $q^{\cdot'''}$ is the heat generation rate per unit time and unit volume, and K is the thermal conductivity. The boundary condition for a cylinder subjected to step surface temperatures T_{s1} for time less than t_1, and T_{s2} for time greater than t_1 is:

$$\begin{aligned} T_{r^*=1} &= T_{s1}\ ; \quad \text{for } 0 < t \leq t_1 \\ T_{r^*=1} &= T_{s2}\ ; \quad \text{for } t > t_1 \end{aligned} \tag{A.6}$$

1. The solution for temperature T_1 in a cylinder with an uniform internal heat source, zero initial temperature, and zero surface temperature is:

$$T_1 = \Phi(1 - r^{*2}) - 8\,\Phi \sum_{n=1}^{\infty} \frac{\psi_n}{\beta_n^2}, \tag{A.7}$$

where $\Phi = \dfrac{q^{\cdot'''} R^2}{4K}$. (A.8)

2. The solution for temperature T_2 in a cylinder with no internal heat source, a prescribed initial temperature distribution, $T_2 = T_{si} + \Phi\,(1 - r^{*2})$, and zero surface temperature, is:

$$T_2 = \sum_{n=1}^{\infty} \left(2\,T_{si} + \frac{8\,\Phi}{\beta_n^2} \right) \psi_n \tag{A.9}$$

3. The solution for temperature T_3 in a cylinder with no internal heat source, zero initial temperature, and step surface temperature boundary condition, $T_3 = T_{s1}$ for $0 < t \leq t_1$, and $T_3 = T_{s2}$ for $t > t_1$, is:

$$T_3 = T_{s1} - 2\,T_{s1} \sum_{n=1}^{\infty} \psi_n \quad \text{for } t \leq t_1, \text{ and} \tag{A.10a}$$

$$T_3 = T_{s2} - 2\,T_{s1} \sum_{n=1}^{\infty} \psi_n - 2(T_{s2} - T_{s1}) \sum_{n=1}^{\infty} \psi_n' \quad \text{for } t > t_1. \tag{A.10b}$$

The final solution, $T_{analytic,\, q\neq 0} = T_1 + T_2 + T_3$, is:

$$T_{analytic,\, q\neq 0} = T_{s1} + \Phi(1 - r^{*2}) - 2(T_{s1} - T_{si}) \sum_{n=1}^{\infty} \psi_n \qquad (A.11a)$$

for $t \leq t_1$, and

$$T_{analytic,\, q\neq 0} = T_{s2} + \Phi(1 - r^{*2}) - 2(T_{s1} - T_{si}) \sum_{n=1}^{\infty} \psi_n - 2(T_{s2} - T_{s1}) \sum_{n=1}^{\infty} \psi_n' \qquad (A.11b)$$

for $t > t_1$.

Effect of Internal Heat Source

The effect of an internal heat source in a cylinder subjected to step surface temperature boundary condition is obtained by subtracting Eq. A.2 from Eq. A.11, i.e.,

$$(T_{analytic,\, q\neq 0} - T_{analytic,\, q=0}) = \Phi(1 - r^{*2}) + 2(T_{si} - T_0) \sum_{n=1}^{\infty} \psi_n$$

$$= [T_{package-steady} - T_0] - (T_{si} - T_0)\left(1 - 2\sum_{n=1}^{\infty} \psi_n\right). \qquad (A.12)$$

Because the parameters β_n, α, t, and R are all positive numbers, the value of the exponential term

$$\exp\left(-\frac{\beta_n^2 \alpha t}{R^2}\right)$$

will always be between zero and one. Therefore,

$$\psi_n = e^{-\left(\frac{\beta_n^2 \alpha t}{R^2}\right)} \left[\frac{J_0(r^* \beta_n)}{\beta_n J_1(\beta_n)}\right] \leq \left[\frac{J_0(r^* \beta_n)}{\beta_n J_1(\beta_n)}\right] \qquad (A.13)$$

and,

$$\sum_{n=1}^{\infty} \psi_n \leq \sum_{n=1}^{\infty} \left[\frac{J_0(r^* \beta_n)}{\beta_n J_1(\beta_n)}\right] \qquad (A.14)$$

Also, from the properties of Bessel functions,

$$\sum_{n=1}^{\infty} \left[\frac{J_0(r^* \beta_n)}{\beta_n J_1(\beta_n)}\right] = \frac{1}{2}. \qquad (A.15)$$

Therefore, $\Sigma\, \psi_n \leq 0.5$, or,

$$\left(1 - 2\sum_{n=1}^{\infty} \psi_n\right) \geq 0. \qquad (A.16)$$

The term $(T_{si} - T_0)$, the steady-state surface temperature rise due to an internal heat source, is always greater than zero. Therefore,

$$[T_{package-steady}(x) - T_0] \geq T_{analytic,\, q\neq 0} - T_{analytic,\, q=0}. \qquad (A.17)$$