Drop Testing of Packages Using Scale Models*

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1. INTRODUCTION

International and U.S regulations require that a package used for transporting radioactive materials maintain containment, shielding and subcriticality of its contents. Regulations require that the package undergo a number of tests under a set of normal and accident conditions (including a number of drop or impact conditions) to demonstrate the package's ability to meet these requirements. The impact conditions involve either (1) the package dropping freely from a height onto a target or (2) a target dropping freely from a height onto a package. Testing large packages is expensive and is often limited by the availability of test facilities. One popular alternative is to test reduced-scale models. This paper examines the feasibility and difficulties of using scale models for the required drop tests.

By analyzing the factors governing the impact responses and failure modes, the paper systematically establishes five general requirements for scale-models used for drop testing. They are the *geometric*, *kinematic*, *dynamic*, *gravitational*, and *material* similarities between the scale model and the prototype. For each of the requirements, the paper identifies the mathematical scaling laws. The impossibility of obtaining a general solution for all the scaling laws is demonstrated. However, by ignoring various effects, the paper produces several practical scale models for various applications. Impact analysis results are used to demonstrate the expected performance of the models. Past applications of the models are identified by citing existing publications.

Besides the fundamental difficulties, the paper also addresses practical difficulties in scale-model testing. Difficulties discussed include the fabrication, inspection, instrumentation, and acceptance testing of a scale model. The practical difficulties impose a lower limit on the usable size of the reduced-scale model. The need for proper interpretation of test measurements and the proper modeling of target characteristics is also emphasized.

The paper shows that the intended purpose of the package drop testing and the expected impact responses and failure modes of the package determine the feasibility and the requirements of scale-model testing. Most of the principles developed here are also applicable to prototype tests and thus should be helpful to both developers and reviewers of drop-test plans for shipping packages. Further details concerning drop testing are given in the original report (Mok et al. 1995).

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2. PAST USE OF THE SCALE-MODEL DROP TEST

Scale models are used because they are often less expensive to produce and test. This statement is particularly true when the test is conducted to obtain performance parameters affected only by a few properties of the prototype. In this case, only a few details need to be simulated, and the test model will cost much less than the prototype.

Past drop tests using a scale model can be grouped by objective into two categories: (1) tests to obtain key data for validating an analysis or an analytical tool which is then used to justify the design of a package (Bumpus et al. 1989, Goldman et al. 1988, Nolan et al. 1989, Robinson et al. 1976); (2) tests to measure structural performance or to demonstrate structural integrity of a package (Huerta 1978 and 1981, Warrant and Joseph 1987). No attempt appears to have been made to use scale models to directly demonstrate the compliance of a package with the regulatory safety-performance requirements. Leak tests on scale models with bolted closures have been conducted following a drop test on those models. However, there exists no indication that the results have been used and accepted as the complete evidence of the compliance of the packages with the regulatory requirement for containment.

The requirements and guidelines developed in this paper are directly applicable to the first two categories of the above-mentioned scale model tests. Tests of the third category (compliance tests) require either the scaling of the safety performance parameters or the correlation of the structural and safety performance parameters. These two tasks are not always possible, because the underlying phenomena for some of the safety performance parameters like leakage rate through the space between two contacting surfaces are not sufficiently understood. Scale-model tests for demonstrating safety performance require special treatments beyond the scope of this paper. In many cases, a scale-model test may not offer sufficient economic advantage to offset the many technical uncertainties. Thus, a full-scale compliance test may remain a preferred choice, as it did for TRUPACT-II (Gregory 1989).

3. SCALE MODEL AND SCALING LAWS

The mechanical response and failure modes of a structure vary with the applied loading, the geometry, material, and temperature of the structure. For example, a compressive load tends to produce buckling failures while a tensile load is more inclined to cause rupture. A slender structure can buckle easier than a stocky structure. A structural component with a notch or with a long internal flaw can fail easier by brittle fracture than a structure without. A material has a greater tendency to deform and to creep at higher temperatures and to fracture at lower temperatures. Therefore, a scale model for predicting the structural response and failure mode must properly simulate the loading, geometry, material, and temperature of the structure. Some radioactive materials have high decay heat; therefore, the simulation of temperature in the scale model is desirable. (The absorption of impact energy in the package can also cause the temperature in the package to rise. However, for the low impact velocity of the regulatory drop test, this effect is highly localized to the area near the impact area and the effect on the total package is negligible.) Unfortunately, the simulation of heat transfer using a scale model is generally considered an impossible task. Therefore, the present discussion omits the simulation of temperatures. It is assumed that the scale model and the prototype have the same temperature distribution or the effects of different temperature distributions can be properly accounted for in the interpretation of the scale model test results.

The standard procedure to develop a scale model and the supporting scaling laws are well established. The necessary processes are as follows:

- Specify the behaviors to be simulated with the scale model
- Understand the physical phenomena causing these behaviors
- Identify the physical parameters controlling these phenomena

- Develop the scaling laws for these controlling parameters by imposing the conditions: (1) The parameters are consistent with each other in dimension; (2) The parameters are similarly related in the scale model as in the prototype.

The last process is commonly known as the dimension analysis (Buckingham 1914 and 1915, Bridgman 1931, Langhaar 1951, Goodier and Thompson 1944, Goodier 1950, Hudson 1961). It consists of two steps:

- Form dimensionless groups of the controlling parameters
- Define the scaling laws in terms of the dimensionless parameter groups

The first step is mathematical and straightforward. The second step depends on the model designer's experience and understanding of the phenomenon to be simulated. The second step is intuitive unless all the governing equations of the phenomenon are known. Then the governing dimensionless groups can be precisely identified from the equations. This precise information usually provides a greater freedom in the design of the scale model; distorted models without complete geometric similarity are possible. Unfortunately, governing equations are known only for a few simple situations.

The use of specific equations is not possible here, because the present discussion focuses on a general package whose design and governing equations are undefined. Thus, the scaling laws must be derived from the general principles which govern the impact response.

In general, to have similar mechanical response, the prototype and the scale model must have:

- Geometric similarity
- Kinematic similarity
- Dynamic similarity
- Gravitational similarity
- Material similarity

These conditions assure that the geometry, the material, the load, the constraint, the impact response, and the governing physical laws are all similar in the prototype and the scale model.

3.1 Geometric Similarity

A complete geometric similarity requires that all geometry-related quantities be scaled using a single scale factor, s_L. A scale factor is defined here as the ratio of a quantity in the scale model to the corresponding quantity in the prototype:

$$s_L = \frac{L_{\text{scale model}}}{L_{\text{prototype}}} \qquad (1)$$

where L is a typical length.

The geometry-related quantities to be scaled with the same factor are:

- geometric dimensions
- locations of impact point
- locations of center of gravity
- locations of measurement sensor

- displacements and deformations

Large deformations, which are likely to occur in a drop test, will appreciably alter the package geometry. Therefore, the geometric similarity will require that the same scale factor be used to scale the geometry and deformation:

$$s_u = \frac{u_{\text{scale model}}}{u_{\text{prototype}}} = s_L \qquad (2)$$

where u denotes a typical displacement or deformation.

The same requirement may also be necessary for small deformations, if the load and constraint patterns and the structural response of the package are very sensitive to deformations. Some examples are: the buckling of a structural component in the package, and the closing of a small gap between two neighboring components.

The use of the same scale factor for deformation and geometry means identical strains in the scale model and the prototype:

$$s_\varepsilon = \frac{\varepsilon_{\text{scale model}}}{\varepsilon_{\text{prototype}}} = 1 \qquad (3)$$

where ε is a typical strain.

This result would restrict the choice of materials for the scale model.

3.2 Kinematic Similarity

This similarity requires that the motions of the prototype and the scale model be similar at homologous locations and homologous times. This requirement implies that the displacements (u), velocities (v) and accelerations (a) of the prototype and the scale model are related by only two scale factors, namely, a time factor (s_t) and a displacement factor (s_u);

$$s_u = \frac{u_{\text{scale model}}}{u_{\text{prototype}}} \qquad (4)$$

$$s_t = \frac{t_{\text{scale model}}}{t_{\text{prototype}}} \qquad (5)$$

$$s_v = \frac{v_{\text{scale model}}}{v_{\text{prototype}}} = \frac{s_u}{s_t} \qquad (6)$$

$$s_a = \frac{a_{\text{scale model}}}{a_{\text{prototype}}} = \frac{s_u}{s_t^2} \qquad (7)$$

Invoking the requirement of geometric similarity, $s_u = s_L$, Eqs. (6) and (7) are rewritten in terms of s_L;

$$s_v = \frac{s_L}{s_t} \qquad (8)$$

$$s_a = \frac{s_L}{s_t^2} \qquad (9)$$

3.3 Dynamic Similarity

This similarity requires corresponding force (F), mass (M), and acceleration (a) to obey Newton's laws in the prototype and in the scale model, i.e.,

$$\left(\frac{F}{Ma}\right)_{\text{scale model}}=\left(\frac{F}{Ma}\right)_{\text{prototype}} \qquad (10)$$

The equation can be expressed in terms of scale factors:

$$s_F = s_M s_a \qquad (11)$$

where s_F, s_M, and s_a are scale factors for the force, mass, and acceleration, respectively.

3.4 Gravitational Similarity (load similarity)

The similarity of force or load between the scale model and the prototype requires that the direction of all forces be unchanged and the magnitude be scaled using a single scale factor. Therefore, the ratio between any two forces will be unchanged from the prototype to the scale model, i.e.,

$$\left(\frac{F_g}{F_i}\right)_{\text{scale model}}=\left(\frac{F_g}{F_i}\right)_{\text{prototype}} \qquad (12)$$

where F_g and F_i represent the gravitational force and the impact inertial force, respectively. For a material particle of a unit volume, the gravitational and impact forces are equal to ρg and $\rho v^2/l$, respectively, where ρ is the mass density of the material; g is the gravitational acceleration; v is the impact velocity and l is the limiter crush; v^2/l is a measure of the peak deceleration of the impact. Thus Eq. (12) becomes:

$$\left(\frac{lg}{v^2}\right)_{\text{scale model}}=\left(\frac{lg}{v^2}\right)_{\text{prototype}} \qquad (13)$$

or in terms of scaling factors,

$$s_u s_g = s_v^2 \qquad (14)$$

The scale factor for the limiter crush is the scale factor for all displacements and deformations (s_u).

3.5 Material Similarity

For a homogeneous material, the mass property is completely defined by its mass density, ρ. The stiffness property, however, is more complicated and is defined by a set of constitutive laws involving stress, strain, and sometimes strain rate. The constitutive laws or the stress-strain relation of many materials including metallic and foam materials can be expressed in the following general form (Marguerre 1962, Drucker 1962, Neilson 1989):

$$G\left(N\sigma/\sigma_o, E\varepsilon/\sigma_o, H\dot{\varepsilon}/\sigma_o, K/\sigma_o\right) \qquad (15)$$

where σ, ε, and $\dot{\varepsilon}$ are the stress, strain, and strain rate in the material, respectively; σ_o is a characteristic or reference stress; N, E, H, and K are material constants defined later in this subsection. The symbol G( ) represents a function of the dimensionless variable groups:

$N\sigma / \sigma_o$, $E\varepsilon / \sigma_o$, $H\dot{\varepsilon} / \sigma_o$, and K / σ_o. The function can be in any mathematical form; it will be a linear function for an elastic material and a non-linear function for an elastic, plastic material. For material of the same behavior, G will have the same mathematical form. Thus, if materials of the same behavior are used for the scale model and the prototype, the constitutive laws of the materials will be similar if the value of each of all the dimensionless variable groups in Eq.(15) is kept unchanged. In addition, the force similarity requires that the ratios σ / σ_o, and σ_o / σ_i remain unchanged between the scale model and the prototype, where σ_i is the impact inertial stress, i.e., $\sigma_i = F_i / A$, and A is the stress area. The resulting conditions for the similarity of the constitutive laws of the material are given in the following equations:

$$\left(\frac{lA\sigma}{\rho v^2}\right)_{\text{scale model}} = \left(\frac{lA\sigma}{\rho v^2}\right)_{\text{prototype}} \qquad (16)$$

$$(N)_{\text{scale model}} = (N)_{\text{prototype}} \qquad (17)$$

$$\left(\frac{K}{\sigma}\right)_{\text{scale model}} = \left(\frac{K}{\sigma}\right)_{\text{prototype}} \qquad (18)$$

$$\left(\frac{E\varepsilon}{\sigma}\right)_{\text{scale model}} = \left(\frac{E\varepsilon}{\sigma}\right)_{\text{prototype}} \qquad (19)$$

$$\left(\frac{H\dot{\varepsilon}}{\sigma}\right)_{\text{scale model}} = \left(\frac{H\dot{\varepsilon}}{\sigma}\right)_{\text{prototype}} \qquad (20)$$

Imposing the geometric-similarity requirements that the same scale factor be used for the geometrical dimensions and the displacements ($s_L = s_u$), and that the strains in the scale model and the prototype are equal ($s_\varepsilon = 1$), Eqs.(16)-(20) can be simplified and rewritten in terms of scaling factors;

$$s_L^3 s_\sigma = s_\rho s_v^2 \qquad (21)$$

$$s_N = 1 \qquad (22)$$

$$s_K = s_\sigma \qquad (23)$$

$$s_E = s_\sigma \qquad (24)$$

and

$$s_H = s_\sigma s_t \qquad (25)$$

Equations (21) through (25) are the scaling laws for the material mass density, ρ and the material constants, N, E, H, and K. The equations show that the scaling of the mass and stiffness properties are related.

For scale-model components where the mass property can be ignored, Eqs.(12) and (14) need not be satisfied. Instead, they are replaced by an equivalent set of equations relating the applied force, F, to the material deformation stress, A σ:

$$\left(\frac{A\sigma}{F}\right)_{\text{scale model}} = \left(\frac{A\sigma}{F}\right)_{\text{prototype}} \qquad (26)$$

or

$$s_\sigma = \frac{s_F}{s_L^2} \quad (27)$$

Equations 22–25 show the scaling of the material constants, N, E, H, and K to be determined by the dimension of the constants. The dimensionless material constant, N, which is like the Poisson's ratio of an elastic material (called the "ratio" constant in this paper), must be kept unchanged between the prototype and the scale model. The "modulus" material constant, E, like Young's modulus of an elastic material, and the "stress" material constant, K, like the yield stress of an elastic, plastic material have the dimension of stress. They, therefore, are scaled in the same way as the stress. Similarly, the "viscosity" material constant, H, representing the material's dependence on the strain rate, must be scaled as a product of the stress and time. The foregoing discussion and equations only demonstrate how each type of the material constants be scaled. They do not address such questions as "What are the constants? "How many constants are needed to completely characterize the material stiffness property?" and "How many are needed to simulate in a scale model?" The answer to these questions depends on the material and the need of the scale model. For example, if a homogeneous, isotropic material is known to behave linear-elastically during a drop test, the stiffness property of the material can be completely characterized using two material constants such as Young's modulus and Poisson's ratio. However, if the impact response is known to depend mainly on Young's modulus, only the modulus will need to be simulated. If the impact generates permanent deformations, then the yield stress and the entire stress-strain curve above yielding should be simulated. As for rate-dependent materials, the strain rate effect is not sufficiently understood to define a general constitutive law like those of the elastic- and rate-independent plastic material. The rate dependence can be more complicated than that presented in Eq.(15). The stress may depend not only on strain rate but also on stress rate, etc. (Lee 1962). Thus, in general, the scaling of rate-dependent material is impossible.

If identical materials are used for the prototype and the scale model, the stress must be kept unchanged for a rate-independent material and both the stress and time must be unchanged for rate-dependent materials. This result can be seen from Eq.(23)-(25) by setting the scale factor of the material constants equal to 1. As shown in the next section, this requirement of keeping both the stress and time unchanged for a rate-dependent material make it impossible to include the simulation of rate effects in such a scale model.

4. DEVELOPMENT OF SCALE MODELS

4.1 Prerequisites

The preceding sections (3.1–3.5) have specified the most general scaling laws or requirements for impact testing. Not all these requirements can be met in a single scale model because of fundamental and practical difficulties, some of which the following section will identify. Fortunately, in many practical situations, not all general requirements need to be met, because part of the impact response and material behavior can be considered insignificant for the situation and the scaling of these behaviors can be ignored. Thus, it is imperative that the developer of a scale model have a general understanding or expectation of the structural and material behaviors of the package to be impact tested.

The impact response can be divided into three responses: wave, vibration, and rigid-body (or quasi-static). Knowing which of the three responses dominates the impact response of the package to be tested is essential to the design of a scale model (see Table 1). Each of the responses depends on different properties of the structure. The properties should be modeled precisely if the response is to be simulated in the model. The rigid-body response, being a steady-state, global response, depends mainly on the total mass of the package and on the force-deformation property of the package near the impact area. Geometry details having no bearing on these properties will be unnecessary to model. On the other hand, the wave response is a transient and localized phenomenon. Its magnitude is affected greatly by the precise distribution of the mass and stiffness of the package as well as the geometry of the reflecting boundaries. Consequently, a model with much greater material and geometry details will be needed for the wave than for the rigid-body response. The vibration response is a global response which also depends on local conditions at boundaries of material and geometry. Similar to the response, the failure modes can also be divided into local and global ones with the local modes requiring greater model details.

Knowing the dominant response in a package is also important to the impact analysis of a package. A traditional method for the analysis is the quasi-static method (Brown 1959, Nelson and Chun 1987). Implicit in the assumption of the method is that the wave and vibration responses can be ignored, and only the rigid-body response is analyzed. However, when the wave and vibration responses are significant, the quasi-static solution will be different from the actual response (Witte 1986) and should be adjusted using a "dynamic load or amplification factor" (Biggs 1964). If a scale model is used to verify the dynamic amplification factor, the model must be designed to also simulate the wave and vibration responses.

The unyielding target used for the regulatory drop test is specified as "a flat and horizontal surface of such a character that any increase in its resistance to displacement or deformation upon impact by the test specimen would not significantly increase the damage of the specimen." (Allen et al. 1987, Gonzales 1986 and 1987.) This requirement implies that the target has no significant wave and vibration responses and behaves mainly like a massive rigid body. Accordingly, the target must have a hard surface, a large mass to minimize surface displacement, and a cubic or cylindrical form to disperse the wave and minimize the wave reverberation and resulting vibration. The target for a scale model should meet these requirements, if the prototype target cannot be used.

4.2 Valid Scale Models and Their Limitations

Since all the parameters used to specify the scale model must have consistent dimensions and there are only three independent dimensions for dynamic mechanical response, i.e., the length (L), time (t), and mass (M) or force (F), there can exist, at the most, three independent scale factors whose value can be arbitrarily chosen. The number of these independent scale factors can be further reduced by theoretical and practical constraints and render the scale model testing impossible. This section shows some of these constraints, and how the constraints can be overcome by ignoring some of the secondary effects in a drop test.

The difficulty in the design of scale model can be shown by examining the requirements for gravitational similarity and material similarity, i.e., Eq.(14) and Eqs.(21)-(25). Since it is difficult to change the gravitational acceleration in a drop test, the gravitational acceleration is usually kept unchanged for the scale model test, that is:

$$s_g = 1 \qquad (28)$$

Thus, the condition for gravitational similarity, Eq.(14) can be satisfied only if

$$s_L = s_v^2 \qquad (29)$$

Since $s_v = s_L / s_t$ is based on dimension consideration, Eq.(29) requires $s_L = s_t^5$. Therefore, the scale factors for time and length can no longer be independently selected.

Similarly, if identical materials are used in the model and the prototype, the scale factors for the mass density, ρ, and all the material constants, N, E, K, and H will be equal to 1 in Eqs.(21) through (25), resulting in the following conditions:

$$s_\sigma = 1 \qquad (30)$$

$$s_t = 1 \quad (31)$$

and

$$s_L^3 = s_v^2 \quad (32)$$

Eq. (32) contradicts Eq. (29). Therefore, if the same material, whether rate sensitive or not, is used in the prototype and the scale model, the gravity effect must be ignored unless the gravitational acceleration can be changed. Eq.(32) also contradicts Eq. (31) unless s_L is set to 1; i.e., unless a full-scale model is used. Since Eq. (31) is needed for strain rate effects, a scale model made of the same material as the prototype will be unable to simulate the strain-rate effect.

The foregoing limitations on the use of identical materials for the scale model and the prototype have long been recognized in the literature of scale-model impact testing. However, since the use of different material to overcome these limitations is difficult and often impossible, almost all scale models use the same material as the prototype and ignore the effects of gravity and strain rate. When using the same material for the scale model, the possible effects of gravity and strain-rate on the measured results must be studied.

Although the use of different material for a scale model is not commonly practiced, the requirements for this procedure are also given in this report. The use of different material may be possible for package components whose material remains elastic during a drop test. If the gravitational acceleration cannot be changed and ignored, the use of different material will be the only alternative.

Table 2 shows all the possible ways to satisfy the scaling laws given by Eqs.(1) through (27). Most of these possibilities rely on some approximations, i.e., neglecting one or more of the following effects: gravity, strain rate, mass, stiffness, wave and vibration. The use of these approximations will provide a greater flexibility in the design of the scale model. The results in Table 2 are presented in terms of scale models for various types of impact response. The "A" models are intended to measure complete impact response including waves and vibrations; the "B" and "C" models are for the rigid-mass and massless material responses, respectively. In each of these model families, several models are presented, using identical or different materials, ignoring gravity or strain-rate effects. For each of the scale models, Table 2 gives the governing scaling laws and the method used to satisfy them; Table 3 gives the scale factors for most of the parameters used in a drop test. The listed scale factors can be easily derived from the independent factors according to the dimension of the test parameters; the inclusion of many of the parameters in Table 3 is merely for convenient reference.

All the scale models described in the literature are included in Tables 2 and 3. Model A-4 satisfies the Scaling Law 2 of Soper (1964); the combination of Model B-2 and C-1 matches the situation described in the same paper for Scaling Laws 1 and 4, and the combination of Models B-2 and C-3 for Law 4. Soper's Scaling Law 2 covers most of the scale models used for dynamic tests with large permanent deformations (Baker 1960, Ezra and Penning 1962, etc.) Almost all the scale models used for package drop tests are identical to Model A-4 (Goldman et al. 1989, Huerta & Yoshimura 1976, Huerta 1978 and 1981, Robinson 1976, Yoshimura et al. 1989, Warrant 1987). A few of the models use Model B-2 for the container and contents, and Model C-3 for the impact limiter (Nolan et al. 1989). No attempt has been made to use different material for the scale model.

To apply the results of Tables 2 and 3 to the design of scale model, it must first be decided which impact response to simulate, then whether the gravity effect can be neglected and whether the material is rate-sensitive. If the wave and vibration responses are significant, one of the Models A must be chosen. If both the gravity and rate effect can be neglected, Model A-4, which uses the same material for the scale model as the prototype, is a good choice.

4.3 Verification of Scale Models

Each of the scale models listed in Tables 2 and 3 is designed to simulate one or more of the three impact responses: the wave, vibration and rigid-body responses. The validity and response characteristics of the models have been demonstrated using the SCANS computer program (Gerhard et al. 1988). The reader can find the detailed results in the original report (Mok et al. 1995).

5. OTHER CONSIDERATIONS IN SCALE-MODEL TESTING

In practice, there are other constraints and considerations for scale model tests than the scaling laws. Some of these considerations are highlighted in this section.

5.1 Environmental/Initial Conditions

An implicit requirement for scale modeling is that the scale model is tested in an environment similar to the prototype's. This means that all environmental conditions such as ambient temperature, solar heat and humidity that are expected to have a significant effect on the property of the package materials and on the initial stress of the package must be properly scaled. The impact response of liquid contents will be drastically changed if the liquid is frozen at low temperature. A wet impact limiter made of wood will perform differently from a dry one. Since it is the mechanical behavior that controls the impact response of a package, the simulation of the environment should be based on the mechanical properties of the package materials. The environment for the scale model must be so adjusted that it produces similar initial mechanical behaviors between the scale model and the prototype.

5.2 Target Characteristics

Like the environmental/initial conditions, the target characteristics must also be considered an integral part of the test model. As discussed earlier in this report, the target behaves as a massive rigid body. In order to satisfy the requirement of "unyielding ground," the target used for a scale-model test must possess a sufficiently large mass, a sufficiently stiff impact area, and a geometry that does not cause significant wave and vibration response.

5.3 Fabrication

The method of fabrication can have a significant effect on the material property and this effect can vary with the size of the fabricated piece. Moreover, in some cases, the fabrication methods used for the prototype and the scale model must be different, because the fabrication method cannot be used for smaller products. For these reasons, it is necessary to identify and evaluate the significance of all possible effects of the fabrication method on the similarity of the models. Joseph (1983) has discussed some of the practical problems in the fabrication assembly and handling of scale models.

5.4 Quality Assurance

The acceptance criteria for the quality assurance program of the scale model should be properly converted from the prototype using the scaling laws. For example, the geometric tolerance and allowable defect size for the scale model must be obtained from the prototype using the scale factor for geometry. The sensitivity and accuracy of the instruments used for inspection should also be adjusted accordingly to assure equal quality between the prototype and the scale model.

5.5 Instrumentation

Like the instruments for quality inspection, instruments for measuring the impact response of the scale model must also have the accuracy and sensitivity to meet the needs of the scale model, not the prototype. The time scale in a reduced-scale model is often shorter than the prototype's. This means that the scale model will have shorter impact duration and higher natural vibration frequencies than the prototype. Thus, the frequency response range of the accelerometer and the instruments for time-history recording and processing must be correspondingly higher for the scale model.

The process of extracting the rigid-body response from the results of a drop test can be cited to demonstrate the need for adjusting the frequency response of data processing equipment. A common method for obtaining the rigid-body response of an impacting package is by filtering the actual time-history records of a drop test (Yoshimura et al. 1989). By using a low-pass or band-pass filter, the wave and vibration responses of a package can be eliminated to provide a fairly accurate time-history record of the rigid-body response. When this technique is used to process the results of a scale model, the natural vibration frequencies of the scale model—not the prototype—should be used to set the cut-off frequency of the filter.

5.6 Statistical Significance of Measurements

For many reasons, a scale model can never be uniformly similar to the prototype. Some of the reasons are the lack of complete knowledge of the underlying phenomena and controlling parameters, and the lack of ability to fabricate the model with the desired results. An example is a bolted joint. The response of a bolted joint depends on many factors, such as preload and surface roughness, that are difficult to measure and control. Therefore, one cannot expect a bolted joint to be reproduced with the same degree of certainty as the total mass of a package. This nonuniformity of the quality of the scale model would cause the test data to show various amounts of scatter. The impact response which depends on less-certain characteristics of the scale model will show greater scatter and less reliability. The measurements of system or global response (like the rigid-body acceleration of the package), would show less scatter than the local response (like the stress in the package), because the global response is affected by parameters of greater control (like the total mass). This difference between the global and local response data is generally supported by published data from drop tests.

Since a greater scatter in the scale-model test data indicates a greater uncertainty in the design of the scale model, and probably also in the prototype design, the scatter data can probably be used to estimate the available margin of safety of the design.

5.7 Size Effects

The microstructure of some materials like honeycomb can be comparable in size to the components of a scale model. In this case, the microstructure will also need scaling. However, this may be impossible because the behavior of a microstructure is usually complex and exhibits a size effect that invalidates scale-model testing.

5.8 Test Confirmation of Scale Model and Scaling Laws

Since the quality of a scale model can be unpredictable, it is essential to obtain direct experimental evidence to validate the scale model and scaling laws. The following are suggestions for obtaining the evidence:

- Measure the total weight and c.g. of the scale model to compare with the prototype
- Apply a load statically and measure the force-displacement relation of the scale model and compare with the prototype
- Measure the natural vibration frequencies of the scale model and compare with the prototype
- Test scale models of at least two different sizes and correlate the data using the scaling laws.

6. GUIDELINES FOR SCALE MODEL TESTING

Summing up the information presented in this report, the following guidelines are suggested for planning and conducting scale-model testing:

Before a scale model can be designed, the objective of the test to be conducted on the model must be clearly specified; i.e., whether it is a test to confirm an analysis or an analysis method, a test to prove the structural integrity of the package, or a test to demonstrate the compliance of the package with the regulatory requirements. The key measurements for the test must then be defined, and their competence in meeting the specified test objective be demonstrated. If it is a test to demonstrate integrity or compliance, the acceptance criteria, in terms of the specified measurements, should also be defined. The objective of the test also determines the complexity and quality control requirements for the scale model. In general, an integrity or a compliance test will require greater details and stricter controls.

To develop the required scaling laws for the specified measurements, the physical phenomena and parameters controlling these measurements must be identified. Depending on the design of the package, the impact response can be dominated by the rigid-body response or also by the wave and vibration responses. The modeling requirement for the rigid-body response is much less demanding. However, for all cases, the stiffness property of the package near the impact point plays an important role and must be well understood and characterized in order to develop the required scaling laws. In general, the condition of geometric similarity, kinematic similarity, dynamic similarity, gravitational similarity, and material similarity should be satisfied everywhere in the scale model. These conditions provide the necessary scaling laws for the desired measurements.

If a different material from the prototype is used for the scale model, the force-deformation behaviors of the material in the drop test must be well understood. The dominant behavior of the prototype and scale-model materials must be similar and governed by a set of definitive constitutive laws characterized by measurable material constants. Scaling laws must be developed for all the influential material constants and used to select the scale-model material. As an example, the elastic behavior of most materials is governed by Hooke's law and characterized by two measurable constants: Young's modulus and Poisson's ratio. Scaling laws have been established here for these two constants and can be used to select a similar material for the scale model.

A scale model cannot be expected to simulate all the influencing phenomena; thus, the omission of some relatively insignificant effects is an acceptable approach. However, the omitted effects must be identified and the possible errors introduced by the omission must be estimated and taken into account in the usage of the results. The effects of gravity and strain rate must be addressed whether the same or different materials are used for the scale model and the prototype. It is highly desirable to use the test data and demonstrate the extent of the omitted effects directly.

All influential details of the scale model must be designed according to the scaling laws. The dimension tolerances of the scale model should be derived from the prototype using the scale factor for geometry. Scaling laws should also be observed to set the environment and test conditions, the target design, the sensitivity of instruments, and the data reduction procedure.

The fabrication methods for the scale model and the prototype must be described and compared. If the same method is used, it is necessary to demonstrate that the property of the finished piece does not vary with its size. If different methods are used, it is essential to show that the property does not vary significantly with the method.

A QA procedure and a set of acceptance standards must be prepared specifically for the scale model. The standards must guarantee similar quality between the scale model and the prototype.

A test procedure must be prepared for the scale model test, specifying the environment and test conditions, the instruments, and the data collection and reduction procedures.

The number of specimens and tests must be sufficient to show the scatter or the statistical nature of the measured data. The requirement is especially important for data that are affected by localized properties which cannot be easily modeled and controlled. The scatter of the data indicates the overall effectiveness and reliability of the scale model test, including the design and fabrication of the scale model. For some cases, the scatter would also be useful for determining the necessary margin of safety for the package design.

Tests should also be carried out using models of different size or scale to provide a direct verification of the scaling laws used for the test.

Table 1. Common Package Design and Expected Dominant Impact Response

PACKAGE DESIGN	EXPECTED DOMINANT IMPACT RESPONSE
Container/contents without impact limiter	All including vibration/wave response
Container/contents with limiter of comparable mass & stiffness	All including vibration/wave response in container/contents and limiter
Stiff container & contents with soft impact limiter	Container/contents: rigid body respons limiter: all including vibration/wave response
Stiff container & contents with light-weight soft impact limiter	Container/contents: rigid body response limiter: massless material reponse

Table 2. Scale Models for Drop Tests

SCALE MODEL	EFFECTS OMITTED IN MODEL	OMITTED SCALING LAWS	APPROACH TO SATISFYING SCALING LAW
SCALE MODELS FOR COMPLETE IMPACT RESPONSE, INCLUDING WAVE AND VIBRATION			
A-1	None	None	• Change material and gravitational acceleration
A-2	Strain-rate effects	• Material similarity in rate-dependent stiffness	• Use same material but change gravitational acceleration
A-3	Strain-rate effects	• Material similarity in rate-dependent stiffness	• Use different material
A-4	Strain-rate and gravity effects	• Gravitational similarity • Material similarity in rate-dependent stiffness	• Use same material
SCALE MODELS FOR RIGID-BODY IMPACT RESPONSE			
B-1	None	• Material similarity in stiffness	• Change gravitational acceleration
B-2	Gravity effects	• Gravitational similarity • Material similarity in stiffness	• Scale factors for length, time, and mass can be independently selected
SCALE MODELS FOR MASSLESS-MATERIAL RESPONSE			
C-1	None	• Dynamic similarity • Gravitational similarity • Material similarity in mass	• Use same material and keep time scale unchanged
C-2	None	Same as C-1	• Use different material
C-3	Strain-rate effects	• Dynamic similarity • Gravitational similarity • Material similarity in rate-dependent stiffness	• Use same material
C-4	Strain-rate effects	Same as C-3	• Use different material

Table 3. Scaling Factors for Scale-Model Impact Test

PARAMETER	PARAMETER DIMENSION	SCALE FACTOR FOR PARAMETER									
		MODEL A-1	MODEL A-2	MODEL A-3	MODEL A-4	MODEL B-1	MODEL B-2	MODEL C-1	MODEL C-2	MODEL C-3	MODEL C-4
BASIC DIMENSIONS											
Length	L	s_L	$s_L = s_g^{-1}$	s_L	s_L	s_L	s_L	s_L	s_L	s_L	s_L
Time	t	$(s_L / s_g)^{.5}$	s_L	$s_L^{.5}$	s_L	$(s_L / s_g)^{.5}$	s_t	1	s_t	s_t	s_t
Mass	$(Ft^2)/L$	s_F / s_g	s_L^3	s_F	s_L^3	s_F / s_g	$(s_F s_t^2)/s_L$	–	–	–	–
Force	F	s_F	s_L^2	s_F	s_L^2	s_F	s_F	s_L^2	s_F	s_L^2	s_F
TEST CONDITIONS											
Drop height	L	s_L	s_L	s_L	1	s_L	s_L	–	–	–	–
Impact velocity	L / t	$(s_L s_g)^{.5}$	1	$s_L^{.5}$	1	$(s_L s_g)^{.5}$	s_L / s_t	–	–	–	–
Impact angle	–	1	1	1	1	1	1				
Accel. of gravity	L / t^2	s_g	s_g	s_g	–	s_g	–	–	–	–	–

Table 3. Scaling Factors for Scale-Model Impact Test, cont'd.

		SCALE FACTOR FOR PARAMETER									
PARAMETER	PARAMETER DIMENSION	MODEL A-1	MODEL A-2	MODEL A-3	MODEL A-4	MODEL B-1	MODEL B-2	MODEL C-1	MODEL C-2	MODEL C-3	MODEL C-4
IMPACT RESPONSE											
Rigid-body Translation	L	s_L	s_L	s_L	s_L	s_L	s_L	–	–	–	–
Rotation	–	1	1	1	1	1	1	–	–	–	–
Trans. velocity	L / t	$\left(s_L s_g\right)^{.5}$	$\left(s_L s_g\right)^{.5}$	$\left(s_L s_g\right)^{.5}$	1	$\left(s_L s_g\right)^{.5}$	s_L / s_t	–	–	–	–
Rotational velocity	1 / t	$\left(s_g s_L\right)^{.5}$	$\left(s_g s_L\right)^{.5}$	$\left(s_g s_L\right)^{.5}$	s_L^{-1}	$\left(s_g s_L\right)^{.5}$	$1 / s_t$	–	–	–	–
Trans. accel.	L / t^2	s_g	s_g	s_g	s_L^{-1}	s_g	$s_L s_t^2$	–	–	–	–
Rotational accel.	$1 / t^2$	s_g / s_L	s_g / s_L	s_g / s_L	s_L^{-2}	s_g / s_L	$1 / s_t^2$	–	–	–	–
Vibration freq.	1 / t	$\left(s_g s_L\right)^{.5}$	$\left(s_g s_L\right)^{.5}$	$\left(s_g s_L\right)^{.5}$	s_L^{-1}	–	–	–	–	–	–
IMPACT RESPONSE—STRESS & STRAIN											
Force	F	s_F	$s_L^3 s_g$	s_F	s_L^2	s_F	s_F	s_L^2	s_F	s_L^2	s_F
Moment or torque	FL	$s_F s_L$	$s_L^4 s_g$	$s_F s_L$	s_L^3	$s_F s_L$	$s_F s_L$	s_L^3	$s_F s_L$	s_L^3	$s_F s_L$
Stress	F / L^2	s_F / s_L^2	1	s_F / s_L^2	1			1	s_F / s_L^2	1	s_F / s_L^2
Strain	–	1	1	1	1	–	–	1	1	1	1

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