

Drop-Test Energy Balance by Computer-Graphics Analysis of High-Speed Videotape*

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INTRODUCTION

This activity was prompted by the concurrent arrival of two items at our Department of Energy Safety Analysis Report for Packaging (DOE SARP) review offices: (i) high-speed videotape of a regulatory 9-m drop test of a shipping package, with the usual cross-hatched white background; and (ii) computer-aided graphics capability. Again we conjectured that the cross-hatched white background was probably provided to allow determination of the package velocities and energies during the portrayed event. But no SARP we had ever seen had made use of this potential.

We also realized that computer graphics would make it easier to produce enlarged frame-by-frame copies of the tape for analysis, than traditional chemical processing of the film. The synergistic outcome was that we decided to attempt to determine by using the tape and a computer, whether the energy E_{dmg} absorbed by damage to the package during the first impact, plus the energy E_{rmg} remaining in the package after the first impact, can be shown to be reasonably equivalent to the drop energy E_{drop} , i.e.,

$$E_{\text{dmg}} + E_{\text{rmg}} \stackrel{?}{=} E_{\text{drop}} \quad (1)$$

The calculation of the three energy terms is discussed in the following sections, preceded by a description of the package subjected to the videotaped drop test, and a calculation of the parallax error involved in the interpretation of the videotaped images.

THE PACKAGE

The outer container of the Advanced Test Reactor (ATR) Fuel Element Shipping Container is a sheet-steel-covered rectangular wooden box with angle-iron edges and 10-cm-thick aluminum-honeycomb ends for added impact limitation (Chappell 1994). Its overall dimensions are $\approx 222 \times 81 \times 28$ cm, and its gross weight is ≈ 3794 N. The container is provided with lifting handles that are described later.

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PARALLAX ERROR

The error x_2 in observed package translation, against the background grid, is determined from the proportionality

$$\frac{x_2}{x_1 + x_2} = \frac{y_2}{y_1}, \quad (2)$$

where x_1 is the actual package translation, y_2 is the distance from the package to the grid, and y_1 is the distance from the camera to the grid. Because $y_1 = 22.9$ m and $y_2 = 3.0$ m during the drop test, the error in observed translation is +15%.

DROP ENERGY E_{drop}

Figure 1, reproduced by computer graphics of a selected frame from the videotape, shows the package when it first touched the rigid slab after being released from its starting position 9 m above the slab. This frame was the first in which the distance the package fell since the previous frame decreased instead of increased. Although the package might appear to remain above the slab, the presence of the hidden lifting handles on the drop edge must be remembered.

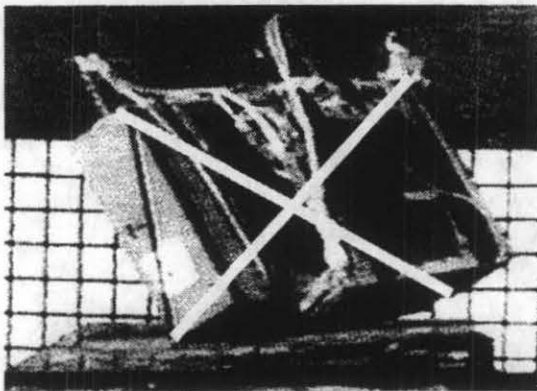


Figure 1. Initial contact during first impact

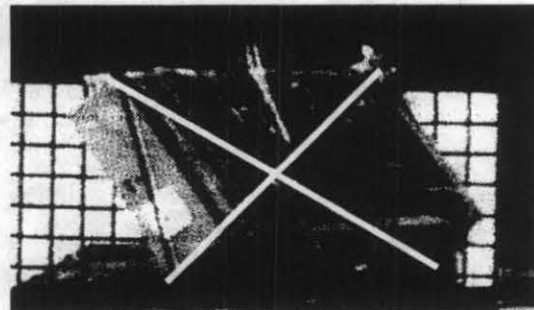


Figure 2. Nadir of CG during first impact

The center of gravity (CG) of the package is indicated by the intersection of the lines drawn between alternate opposite corners. Figure 2 shows the nadir of the package CG between its first and second impacts with the slab. Determination of the specific nadir frame was aided by making marks on the monitor screen to record the movement from frame to frame of the upper surface of the package.

The position of the CG in Figure 2 scales to be ≈ 15 cm lower than it is in Figure 3, where the distance corresponds coincidentally with the spacing (6 in.) of the grid lines on the white background. Thus the drop energy was

$$E_{\text{drop}} = 3794(9.15) = 34,700 \text{ J.} \quad (3)$$

The movement of the CG from Figure 1 to Figure 2 also enables estimation of the deceleration load on the package contents.

ENERGY ABSORBED BY PACKAGE DAMAGE E_{dmg}

To demonstrate compliance with the regulatory 9-m drop test requirement, the package was dropped on a long edge on which were two 66-cm-long, 10-cm-dia., 60° reinforced pipe-sector lifting handles. The handles were welded to the 5.1 x 0.3-cm edge angle iron, ≈ 15 cm apart. As a result of the drop impact, the concentrated load on the projecting handles permanently twisted the angle iron at and between the handles $\approx 45^\circ$, as estimated from inspection of a still photograph taken of the package after the test, Figure 3.

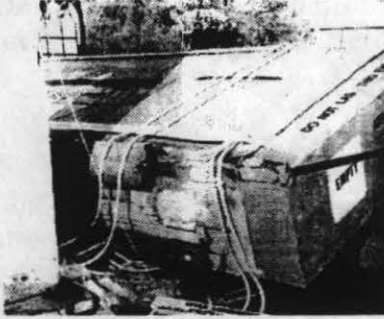


Figure 3. Damage from test

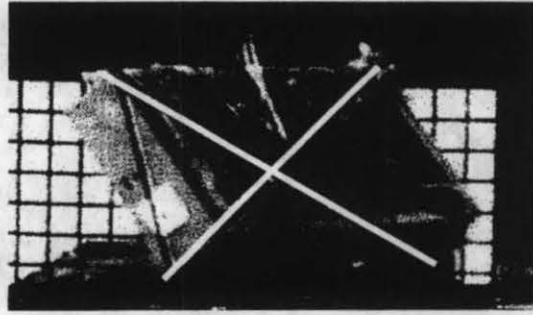


Figure 4. Apex of CG between first and second impacts

To obtain the energy absorbed by the twisting of the angle iron, the torque required to cause the twisting is needed. This torque was estimated by using the "sand-hill analogy," which assumes perfect, non-strain-hardening plasticity (Flugge 1962). This analogy affirms that the volume of sand going into a hill supported by a transverse cross-sectional replica of a twisted body, is proportional to the torque required to produce the twist. The proportionality is related to the slope α of the hill, which can be found by considering the twisting of a cylinder, for which the torque is known (Turula 1995).

The height $H(r)$ of a sand hill on the circular cross-section of the cylinder equals $(R - r)\alpha$, where R is the radius of the cylinder, r is the radial position of $H(r)$, and α is the slope of the hill. The volume V_{cyl} of the hill is

$$\begin{aligned} V_{cyl} &= 2\pi \int_0^R r H(r) dr \\ &= \frac{\pi}{3} \alpha R^3. \end{aligned} \quad (4)$$

The torque T_{cyl} required to twist the cylinder is

$$\begin{aligned} T_{cyl} &= 2\tau \int_0^{\pi R} \int_0^{\theta} r^2 dr d\theta \\ &= \frac{2}{3} \pi \tau r^3, \end{aligned} \quad (5)$$

where τ is the shear strength and θ is the angle of twist.

Because V_{cyl} and T_{cyl} are equivalent by analogy,

$$\alpha = 2\tau. \quad (6)$$

Turning now to the actual angle iron, the maximum height of its sand hill is $\alpha t/2$, where t is the thickness of the angle. Also, this maximum height extends to within $t/2$ of the tip of the angle. Thus the volume V_1 of the hill on both legs, from the inside corner to within $t/2$ of each tip, is

$$V_1 = \frac{1}{2} \left(w - \frac{3}{2}t \right) \alpha t^2, \quad (7)$$

where w is the outside width of each angle leg. The remaining volume of the sand hill is composed of pyramids or prisms with volumes $A_b h/3$ and $A_b h$, respectively; A_b is base area and h is height. Summarizing,

Location	Type	A_b	h	No. of Identical Volumes	Volume
Tip	Pyrmnd	$t^2/2$	$\alpha t/2$	2	$2(t^2/2)(\alpha t/2)/3 = \alpha t^3/6$
Corner	Pyrmnd	$t^2/8$	$\alpha t/2$	2	$2(t^2/8)(\alpha t/2)/3 = \alpha t^3/24$
"	Pyrmnd	$\alpha t^2/4$	$t/2$	2	$2(\alpha t^2/4)(t/2)/3 = \alpha t^3/12$
"	Prism	$\alpha t^2/8$	$t/2$	2	$2(\alpha t^2/8)(t/2) = \alpha t^3/8$
Total:					$V_2 = \frac{5}{12} \alpha t^3$

Then, the total volume V_{ang} of the sand hill angle is

$$V_{ang} = V_1 + V_2 = \left(\frac{1}{2}w - \frac{1}{3}t \right) \alpha t^2. \quad (9)$$

Because $\alpha = 2\tau$, the torque required to twist the angle is

$$T_{ang} = \left(w - \frac{2}{3}t \right) \tau t^2 \quad (10)$$

and the energy required to twist the angle θ radians in two places is

$$\begin{aligned} E_{ang} &= 2T_{ang}\theta \\ &= 2 \left(5.1 - \frac{2}{3}0.3 \right) \frac{2.76 \times 10^8}{10^6} 0.3^2 \frac{45}{180} \\ &= 60.8 \text{ J}, \end{aligned} \quad (11)$$

where 2.76×10^8 pascals is the shear strength of the angle.

Additional energy was absorbed by crushing and/or shear of the fir plywood box behind the angle iron. It is assumed that the crushing and shear energies are additive. The crushing energy E_{cr} is

$$\begin{aligned} E_{cr} &= \sigma_{cr} V_{cr} \\ &= \frac{6.00 \times 10^6}{1 \times 10^6} \pi (5.1 - 0.3)^2 \frac{45}{360} [2(66) + 15] \\ &= 7980 \text{ J}, \end{aligned} \quad (12)$$

where σ_{cr} is the crushing strength of the fir, 6.00×10^6 Pa, and V_{cr} is the volume of fir crushed. The shear energy E_{sh} is

$$E_{sh} = \frac{1}{2} \sigma_{sh} \int_0^{A_T} d_{sh} dA, \quad (13)$$

where σ_{sh} is the shear strength of the fir, 8.00×10^6 Pa, d_{sh} is the shear distance, and A_T is the total area sheared. The 1/2 factor is the same as that is used for estimating punch press energy consumption. For the two ends of the depression in the fir, $d_{sh} = r\theta$ and $dA = r\theta dr$, whereas for the long side of the depression, $d_{sh} = R\theta$ and $A_T = R\theta L$, where R is the width of the depression and L is the length. Therefore,

$$\begin{aligned} E_{sh} &= 2 \frac{1}{6} \sigma_{sh} \theta^2 R^3 + \frac{1}{2} \sigma_{sh} \theta^2 R^2 L \\ &= \sigma_{sh} \theta^2 R^2 \left(\frac{1}{3} R + \frac{1}{2} L \right) \\ &= \frac{8.00 \times 10^6}{1 \times 10^6} \frac{45^2}{180} \pi (5.1 - 0.3)^2 \left\{ \frac{1}{3} (5.1 - 0.3) + \frac{1}{2} [2(66) + 15] \right\} \\ &= 6400 \text{ J} \end{aligned} \quad (14)$$

Thus, the energy absorbed by package damage on the first impact is

$$E_{dmg} = E_{ang} + E_{cr} + E_{sh} = 14,400 \text{ J}. \quad (15)$$

ENERGY REMAINING IN PACKAGE E_{rmg}

Figure 4 shows the apex of the package CG between the first and second impacts. Because there is essentially no difference in elevation between the nadir and apex positions of the CG, no potential energy $E_{rmg,pot}$ remained in the package after the first impact.

Figure 5 displays the package as it became horizontal between the first and second impacts, and Figure 6 shows the package when it first touched the slab again after the first impact had occurred. To find the rotational energy remaining in the package, $E_{rmg,rot}$, after the first impact, first the visible ends of the package in both Figures 3 and 6 were rotated by

manual graphics into the plane of the paper to establish that the package rotated $\approx 82^\circ$ about its longitudinal axis between the first and second impacts.

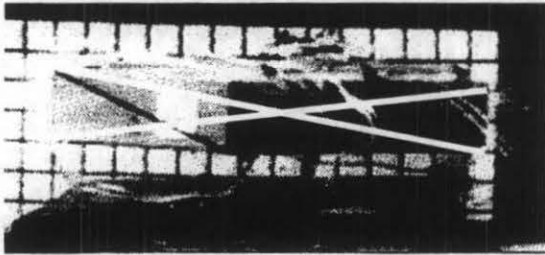


Figure 5. Horizontal position between first and second impacts

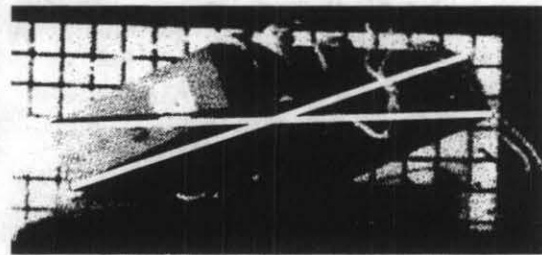


Figure 6. Initial contact during second impact

The elapsed time between impacts, determined by averaged stop-watch measurements as corrected for the ratio between tape viewing and recording speeds, was ≈ 0.086 s. Thus, the angular velocity ω of the package between impacts was

$$\omega = \frac{82}{180} \frac{1}{0.086} = 5.3 \text{ rad/s.} \quad (16)$$

The polar moment of inertia I_p of the package about its longitudinal axis was

$$I_p = 3794 \frac{\overline{81}^2 + \overline{28}^2}{12(1 \times 10^2)9.80} = 23.7 \text{ n-m-s}^2. \quad (17)$$

Thus,

$$E_{\text{rmg,rot}} = \frac{I_p \omega^2}{2} = 665 \text{ J.} \quad (18)$$

To find the translational energy remaining in the package $E_{\text{rmg,tra}}$ after the first impact, we scaled the distance traveled by the CG between Figures 2 and 6 and found it to be 8.8 cm. Thus, the translational velocity of the package between impacts was

$$v = \frac{8.8}{0.086} = 103 \text{ cm/s} \quad (19)$$

and

$$E_{\text{rmg,tra}} = \frac{mv^2}{2} = \frac{3794 \overline{103}^2}{2 \overline{100}^2 9.80} = 204 \text{ J.} \quad (20)$$

CONCLUSION

It is now possible to answer the question raised in the Introduction:

$$E_{\text{dmg}} + E_{\text{rmg}} \stackrel{?}{=} E_{\text{drop}}. \quad (1)$$

Specifically,

$$14,400 + (665 + 204) \stackrel{?}{=} 34,700 \quad (21)$$

Obviously, the drop energy exceeds the sum of the other calculated energies by the very large amount of 19,400 J. (The more favorable outcome reported in the Abstract was based on calculational procedures that have been since improved.) We believe that the missing energy was absorbed by the crushing of wood, which is not readily deducible from Figure 3. It is logical that when the package descended 15 cm from its first contact to its nadir, during the first impact, elastic deformation of the angle iron and sheet steel was accompanied by crushing of wood and honeycomb along the entire length of the drop edge. This conclusion is supported by Figure 3, which does show the honeycomb was crushed along the drop edge.

To find the average diagonal depth d the wood and honeycomb in the drop edge that must have been crushed, the missing energy E_{mis} can be equated to the product of the crushing strength and volume of the wood and honeycomb crushed:

$$E_{\text{mis}} = \sigma_{\text{cr}} L d^2 \quad (22)$$

$$19,400 = \frac{6.00 \times 10^6}{1 \times 10^6} 222d^2.$$

In Eq. 22, the crushing strength of the honeycomb is assumed reasonably to be the same as that of the wood. Also, d^2 equals the cross-sectional area of the crushed material because d is the altitude of an isosceles triangle having base angles of 45° . From Eq. 22, to account for E_{mis} the average crushing depth d must have been some 3.8 cm, which is an entirely possible amount.

The foregoing procedure assumes that the energy absorptions, from crushing along the entire drop edge and crushing and shear caused by twisting of the drop-edge angle iron, are additive. If the wood energy absorptions due to the angle iron twisting are completely disregarded, the average depth d necessary for an energy balance, from Eq. 22, is increased to a still reasonable value of about 5.0 cm. The actual value of d cannot be verified directly because the wood burned up during the hypothetical accident thermal test that followed the drop test. The package contents did not appear to lose containment as a result of the drop and thermal tests.

This analytical procedure may be of value when packages that fail the drop test are redesigned, and when the margin of safety of a package provides under hypothetical accident conditions is estimated. The procedure could be automated, but we believe that development of the necessary software is not cost effective.

ACKNOWLEDGMENT

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