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Uncertain to Unambiguous--Material Balance Evaluations			
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The purpose of this document is to briefly outline the general methodology of the standard error of the inventory difference (SEID) calculations with emphasis on elements of the calculation that may be overlooked by personnel new to the concepts trying to develop the calculation. This document applies to facility personnel who may be interested in developing or cost-scheduling a project to develop such a calculation as well as other stakeholders such as corporate or regulatory oversight personnel interested in evaluating or encouraging the development of such statistical models and calculations.			
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1.0 INTRODUCTION

This paper is intended to facilitate discussions regarding the need for and potential cost-benefit (or risk reduction) of performing statistical evaluations of individual material balances as well as historical trend analyses. Other analyses corollary to these evaluations is an understanding of active inventory as well as individual measurement systems and material types that pose the most risk to the SNM accountancy system.

1.1 Purpose, & Scope

The purpose of this document is to briefly outline the general methodology of the standard error of the inventory difference (SEID) calculations with emphasis on elements of the calculation that may be overlooked by personnel new to the concepts trying to develop the calculation. This document applies to facility personnel who may be interested in developing or cost-scheduling a project to develop such a calculation as well as other stakeholders such as corporate or regulatory oversight personnel interested in evaluating or encouraging the development of such statistical models and calculations.

1.2 Limits of Applicability

This document is not a comprehensive discussion of the statistical methodology of the calculation of the SEID, individual inventory difference evaluations, or inventory difference trending. It should not be used as the sole resource for completing such a calculation at a specific facility, but as a guide to finding resources and planning for such a development. This document does not propose to comply with the regulations or requirements of any location or facility.

2.0 SUMMARY OF TERMS & DEFINITIONS

Most material control and accounting definitions are interchangeable among facilities using the language of IAEA, EURATOM, and US Nuclear Regulatory Commission (USNRC) regulations, handbooks, or guidance documents. For example, special nuclear material (SNM) and material balance area (MBA) are two terms commonly in use by nuclear industry personnel around the world. This section simply attempts to relate a few USNRC terms to those used by the IAEA in order to facilitate a clear understanding among a wide variety of readers.

A. Inventory Difference (ID)

This term may be used interchangeably with material unaccounted-for (MUF). It is the difference between what is expected to be on-hand at a physical inventory (book inventory) versus what is physically found (physical inventory). The book inventory is obtained by

Book Inventory = Beginning Inventory + Additions – Removals

Where the beginning inventory is the previous inventory which marks the beginning of the material balance period. The additions are additions to process, that is, receipts.

Removals are shipments or measured discards whose current-period measurements update accounting records.

The physical inventory is the most recent inventory which marks the end of the material balance period. In a bulk special nuclear material (SNM) processing facility, the only time the book inventory will match the true material balance on-site is if the material balance is corrected for the inventory difference as a miscellaneous shipment. The inventory difference is

$$\text{Inventory Difference} = \text{Book Inventory} - \text{Physical (Ending) Inventory}$$

Or, more specifically

$$\text{ID} = \text{Beginning Inventory} + \text{Additions} - \text{Removals} - \text{Ending Inventory}$$

Thus, an evaluation of the inventory difference (or MUF) is an evaluation of our capability to calculate accurate and precise ending material balances. In a nuclear material accountancy system that is performing as it should, the expected value of the inventory difference is zero. This means that over many material balance periods, the sum of the inventory differences should approximate zero, and that for any individual period, the inventory difference should not be statistically distinguishable from zero.

B. Standard Error of the Inventory Difference (SEID)

This term may be used interchangeably with σ-MUF. It represents the total, propagated measurement uncertainty of the inventory difference. It accounts for both systematic and random variances. The ID, and thus the SEID, are sensitive only to those inventory items, additions to process, and removals from process whose measurements or remeasurements updated nuclear material accounting records during the material balance period.

These two concepts—random and systematic uncertainties as well as the sensitivity of the ID only to special nuclear material that was measured during the period—are key in an appropriate SEID propagation.

$$\text{SEID} = \sqrt{\sum_{\text{MBC}}^{\textit{All MBCs}} \sum_{\text{STRATUM}}^{\textit{All Strata}} \text{C}^2_{\text{Random}_{\text{STRATA}_{\text{MBC}}}} \cdot \left(\text{RSD}^2_{\text{RANDOM}_{\text{Meas systems}}}\right) + \sum_{\text{STRATUM}}^{\textit{All Strata}} \left(\text{C}_{\text{Sys}_{\text{BI}}} + \text{C}_{\text{Sys}_{\text{A}}} - \text{C}_{\text{Sys}_{\text{R}}} - \text{C}_{\text{Sys}_{\text{EI}}}\right)^2 \cdot \left(\text{RSD}^2_{\text{Sys}_{\text{Meas systems}}}\right)}$$

C. Material Balance Period

This is sometimes called an inventory period. It marks the dates on a calendar between which a material balance is calculated, corrected for the inventory difference and reconciled (for grams U and ^{235}U, obligations, material type blending, etc.) between the facility and regulatory authorities through a central reporting database. Financial accounting normally follows this since SNM is likely the highest valuable commodity possessed and used by the facility. In the US, commercial fuel cycle facilities licensed by the NRC to process uranium enriched to $\leq 5\%$ ^{235}U, this is normally 1 calendar year plus or minus 30 days unless a special inventory is required at a shorter frequency.

D. **Material Balance Components**

This simply refers to the mathematical terms used to calculate the inventory difference. They are the beginning inventory, additions to process (receipts), removals from process (shipments and measured discards), and ending inventory. Overall, the concept of inventory strata is nested within material balance components, which are nested within a material balance area. If only a few material types are received or shipped or discarded, all inventory strata can be represented within each material balance component within each material balance area without changing the overall inventory difference or SEID calculation results.

E. **Active Inventory**

The total SNM mass of all items, batches, or lots that could have contributed to the ID by receiving a measurement during the current material balance period. Active inventory is how the SEID is evaluated—made relative to the active inventory and compared to a regulatory or other limit, such as 0.125% of the active inventory required by the USNRC for commercial fuel cycle facilities. Thus, a facility is required to control the overall variance of the inventory difference by controlling individual measurement systems or mix and sampling systems. If a change degrades one measurement system's capability, then the amount of the active inventory measured by that system can be used to determine the potential impact to the overall accountancy system. Reference [1] indicates that the active inventory is superior to throughput (larger of additions or removals) for evaluating material balances and even evaluating the tools used to evaluate material balance.

F. **Measurement System**

A measurement system is the combination of method, material, matrix, and machine used to describe how a certain material type obtains a measured attribute within its inventory stratum. For example, a bulk method normally results in a net weight of material. This bulk system may be one level indicator and volumetric calibration curve and laboratory density method associated with one bank of uranyl nitrate tanks, or it may be a type of weighing device calibrated to weigh pellets on trays across all production lines using the same size and type of tray and measurement control protocol. Conversely, it could be one titrator that measures various material types in various chemical/physical forms from various sampling systems across the plant—each material type having their own "system" designation.

G. **Inventory Stratum**

As it is used in this document, an inventory stratum is defined by a unique combination of measurement systems used to obtain grams ^{235}U within each material balance component and material balance area.

Figure 2.5-1: Tabular Representation of the Relationship Between Strata, MBCs, & MBA

MBA 1

MBC 1 (Beginning Inventory)			
Stratum 1	gU_{BI}	$gU235_{BI}$	B-1; S-1; E-1; I-1
Stratum 2	gU_{BI}	$gU235_{BI}$	B-2; S-2; E-1; I-1
Stratum 3	gU_{BI}	$gU235_{BI}$	B-3; S-2; E-2; I-1
⋮	⋮	⋮	⋮

MBC 2 (Additions)			
Stratum 1	gU_{A}	$gU235_{A}$	B-1; S-1; E-1; I-1
Stratum 2	gU_{A}	$gU235_{A}$	B-2; S-2; E-1; I-1
Stratum 3	gU_{A}	$gU235_{A}$	B-3; S-2; E-2; I-1
⋮	⋮	⋮	⋮

MBC 3 (Removals)			
Stratum 1	gU_{R}	$gU235_{R}$	B-1; S-1; E-1; I-1
Stratum 2	gU_{R}	$gU235_{R}$	B-2; S-2; E-1; I-1
Stratum 3	gU_{R}	$gU235_{R}$	B-3; S-2; E-2; I-1
⋮	⋮	⋮	⋮

MBC 4 (Ending Inventory)			
Stratum 1	gU_{EI}	$gU235_{EI}$	B-1; S-1; E-1; I-1
Stratum 2	gU_{EI}	$gU235_{EI}$	B-2; S-2; E-1; I-1
Stratum 3	gU_{EI}	$gU235_{EI}$	B-3; S-2; E-2; I-1
⋮	⋮	⋮	⋮

3.0 ASSUMPTIONS & INPUTS

3.1 Assumptions

There is a plethora of statistical assumptions that underly any statistical analysis of the size and complexity of the SEID. Reference [1] would be the place to begin to apply statistical concepts to the SEID of a given location where evaluations can be made regarding the nature and validity of those assumptions. This section attempts to generally outline the major basis of the limit calculations (or equivalently, hypothesis tests).

That basis is the assumption—or perhaps assertion—that measurement uncertainty is the only contributor to inventory differences. This assumption is important in flagging non-measurement ID contributors. If human error, unmeasured losses, uncorrected measurement biases, or malicious diversions occur, these would be unaccounted-for ID contributors and would also be unaccounted-for in a measurement uncertainty propagation, giving them higher probability of being flagged by a test or limit calculated using only measurement uncertainties.

3.2 Inputs

3.2.1 <u>Statistical Methodological Guidance</u>

The USNRC statistical guidance is a publication called NUREG/CR-4604, “Statistical Methods in Nuclear Material Accounting” (Reference [1]). It was commissioned by the NRC and generated by Bowen and Bennett of Pacific Northwest Laboratory (PNL) in 1988. It covers many foundational statistical concepts then moves into concepts specific to nuclear material accountancy—from error modeling to determination of individual measurement system uncertainties to error variances for strata to the calculation of the SEID, active inventory, and historical trending. It also includes other helpful topics such as variable and attribute sampling plans as well as measurement control strategies. Specifically, chapters 14 and following are most relevant to the discussion in this document.

3.2.2 Regulatory Requirements

Facilities around the world follow various local, regional, and international safeguards regulations and guidance handbooks. In the US, the applicable regulation is Title 10, Code of Federal Regulations part 74 (Reference [2]). These regulations further delineate between facilities who process highly enriched uranium and those who process commercial grade uranium. For the commercial fuel cycle facilities within the scope of this document, 10 CFR 74.31 specifies the use of the SEID as a loss detection tool. A facility-specific goal quantity is established as a function of grams ^{235}U throughput. The detection threshold is established as a 90% lower confidence limit on that goal quantity using the SEID in grams ^{235}U. However, most facilities also establish two- or three-sigma limits around the expected U and/or ^{235}U ID of zero and use a graded approach for problem identification and investigation of IDs that exceed these thresholds. These limits tend to be somewhat smaller than the NRC-established detection threshold.

4.0 CALCULATION DISCUSSION

This document does not contain a detailed calculation of an SEID. Each SEID calculation is very specific to the operation and nuclear material accountancy system of the subject facility and includes sensitive information. However, a discussion of the most important considerations is still helpful especially in understanding terms and definitions.

4.1 SEID Statistical Methodology Overview & Data Requirements

Within a material balance area (or whole plant), there are a two main strategies for the calculation of the SEID that may be employed, an additive model based on absolute uncertainties and a multiplicative model based on relative uncertainties. This paper discusses the latter, which seems to be the most popular among USNRC licensee facilities.

Both strategies involve stratifying the material balance area’s (or facility’s) inventory into unique bulk, chemical, and isotopic properties. This document recommends using each unique combination of measurement methods to stratify the inventory. This provides measurement points that may include the bulk, sampling, element analysis, and isotopic analysis (for ^{235}U SEID) measurement systems associated with each nuclear material accountancy record used to calculate the material balance. However, these methods will have one-sigma uncertainties associated with them in their original units, whereas the desire is to propagate a variance of the inventory difference in terms of grams ^{235}U, for example.

The multiplicative model’s approach is to make each of these uncertainties relative to the most relevant value—examples: for systematic scale error standard deviations, make them relative to the certified value of the standard; for non-destructive assay, calculate replicate differences relative to the measured value used on the accounting record, whether it is the original measurement or the average of several measurements. These unitless variances may now be multiplied by sensitivity coefficients in the desired units—

usually grams uranium or grams ^{235}U isotope. In this way, the grams ^{235}U measured by that system in that strata within that material balance component are used to quantify a specific relative standard deviation's (RSD) impact to the accountancy system as described in the random and systematic coefficients discussions below.

Now that these relative random and systematic variances are stated in the desired units, sum them across any desired array, whether it be across all measurement systems, across an inventory stratum, or among only certain measurement systems of a certain type.

Thus, there are several main components to the calculation of the SEID: Inventory stratification, material balance components, systematic error variances, systematic sensitivity coefficients, random error variances, random sensitivity coefficients, and finally the combination of these into a variance of the ID and standard error of the inventory difference.

4.1.1 Material Balance Area & Material Balance Period

Most operators of fuel cycle facilities understand the concept of breaking their processes up into material balance areas—at the very least defining their bulk SNM operations as one contiguous material balance area. Most requirements specify that it be a contiguous area, but others may only require that it be assigned custodial responsibility while following the transactional requirements for receipts, shipments, material balances etc. Most if not all SNM processing facilities have regulatory requirements dictating the longest inventory frequency and require at least one MBA for accounting purposes.

The concept is to isolate any flagged material balance, material control, item control, or measurement control issues in time and space. The MBA draws a box around the problem in space, and the MBP draws a box around the problem in time. Smaller MBAs and shorter MBPs facilitate more specificity, but the cost in administrative burden quickly becomes prohibitive.

A. **Data Requirements**

The material balance area is the base unit for conducting and evaluating material balances. USNRC requires a plant-wide inventory difference calculation and evaluation, and therefore some facilities will calculate the ID and SEID treating the entire plant operation as one MBA, but, internally they also separate the plant into multiple MBAs and similarly calculate and evaluate by MBA. For a single MBA, the additions, removals, and inventories must be measured in such a way as to ascertain the standard deviation of each measurement—whether specifically or more frequently by a modeled uncertainty using the previous material balance period's SEID uncertainties. Even so, measurement control protocols are implemented, control standard measurements, calibrations, replicate measurements, and responses to measurement control events are required. Likewise, the input and output measurements must be controlled, evaluated, and biases or changes to the precision detected and corrected. Moreover, transactions must show these input and outputs as well as inventory measurements, which can become burdensome to database administrators and operators alike. These calibration, control, replicate, event identification/response, and transaction data must be recorded on a system robust against human error and falsification in such a way as to be retrievable for the SEID calculation.

B. **Data Requirements and Material Balance Components**

The material balance data will normally come from one record source for receipts and shipments—and possibly yet another for additions and removals from MBAs that cover only internal processing—and another type for the inventory listing. So, the ability to query each data base or record source into one place for further manipulation and calculation is needed to calculate the inventory difference, the sensitivity coefficients, and the active inventory. See the discussions below on sensitivity coefficients and active inventory to see what details are needed and how the data is used.

4.1.2 Active Inventory & Data Requirements

The active inventory is taken from the same data that provides systematic coefficients. The user must know which material balance components on each item appears, which is easily done using the record source (inventory table, shipping/receiving records, etc.). For the SEID coefficients, it is not adequate to simply total grams of SNM added, removed, and inventoried. One must be able to trace batches and lots to their laboratory management system results. Additionally, one must be able to determine if the item, batch, or lot received a measurement during the inventory period.

For example, each rod within an assembly—or in any other container which a rod may be inventoried, shipped, or received—must be identified to ensure that it is part of the active inventory. If the rod was inventoried on the beginning inventory but was shipped to the customer using the exact same grams U and grams ^{235}U, then this rod did not contribute to the inventory difference and is therefore not part of the active inventory. Only active inventory items, batches, or lots could contribute to the inventory difference, and only those should be used to calculate the SEID and evaluate the inventory difference whereas all the items inventoried, received or shipped were used to calculate the original material balance reported to the regulatory authority.

The rod in the example above is called a common term, which is defined as the same item ID found on opposing material balance components using the equivalent formulation for inventory difference below. Beginning Inventory and Additions are on opposite sides of the subtraction sign from Removals and Ending Inventory. So, if an item is listed on the same SNM values on beginning inventory and shipments (or additions and ending inventory; or beginning and ending inventories), the subtraction cancels this items' SNM masses in the material balance, and should be removed as a sensitivity component since the inventory difference was not sensitive to this measurement system's uncertainty.

$$ID = \mathbf{(BI + A)} - \mathbf{(R + EI)}$$

All such items must be identified by the query. A test of an SEID model for this paper yielded a ~150% overestimation of the active inventory and nearly that much in the ^{235}U SEID when common terms were not eliminated. This would have a significant impact on decision rules and decisions regarding the overall health of an SNM accountancy system's capabilities. Overconfidence in the capability of the SNM accountancy system results from overstated variances (due to sensitivity components) that appear to be much smaller relative to the overestimated active inventory versus correctly calculated coefficients.

This is the most difficult part of the SEID calculation. Uncertainty data may be manually entered and calculated, but manual calculation of sensitivity coefficients is labor intensive and prohibitive given the tight time constraints of most inventory reconciliation requirements. Once this data—item by item and batch by batch—is collected and duplicates flagged, the systematic coefficients are taken directly from this data in linear terms and the random coefficients are the squared SNM masses for each item. Downstream processing is fairly straight forward as described below.

4.1.3 Inventory Stratification & Measurement Systems

After the collection of all items and SNM masses, these can be labeled according to a previously established inventory stratification. The SEID model used for this document numbers the strata so that they can easily be codified. The inventory table generated each MBP sorts each material designation in the plant into inventory codes, which are on a database lookup table that specifies sampling and other inventory requirements—including exclusion from inventory such as scrap pellets of unknown enrichment or other hard-to-sample material types.

So, each material designation has an inventory code, and inventory codes are further aggregated into SEID codes, which are the strata. These SEID codes are assigned to the items inventoried on the inventory table, so the active inventory query pulls that directly. Then, detail lines for receipts, shipments, and discards are sorted into these codes via lookup table similarly to the inventory table, which could easily be accomplished using EURATOM or IAEA composition codes from inventory listings or inventory change records.

Each stratum is a stratum because it has a unique combination of bulk, sampling, element, and isotopic measurement systems. Thus, these systems are assigned to each stratum. Many strata may have the same element and isotope analytical methods, but may differ in sampling method based on chemical or physical form or in bulk measurement method based on container size.

4.1.4 Systematic Error Variances and Relative Standard Deviations (RSDs)

The systematic error variance of a measurement system typically includes the major bias contributors for that system. For a given measurement system (*i*), these tend to be calibration error (or the uncertainty of the standard certification, S_0, for systems with point calibrations which are usually reported at $2 \cdot \sigma$), readability (Δ) error variance, and the standard error of all in-control standard measurements from the measurement control program.

$$\sigma_{\text{sys}_i} = \sqrt{\frac{s^2_{\text{stds}_i}}{n_{\text{stds}_i}} + \left(\frac{S_{0_i}}{2}\right)^2 + \frac{\Delta_i^2}{12}}$$

The systematic variances may be pooled over multiple standard types within a single measurement system and/or pooled across like-kind measurement systems. For example, the high and low control standard measurements (and readability, etc.) across all pellet scales may constitute σ_{sys} for pellet tray weighing.

4.1.5 Systematic Sensitivity Coefficients

Once the active inventory is divided into its strata among the material balance components, the sensitivity coefficient for each measurement system's systematic relative variance is that stratum's inventory difference. This is because bias does not play a role in a process whose inputs and outputs are measured on the same measurement system using the same calibration curve regarding the calculation of the additions to process or removals. For example, inventory strata whose SNM masses are only measured for inventory (EURATOM or IAEA alpha KMPs) and whose magnitude is very similar—albeit with newly measured items each year—the systematic uncertainty only applies to the difference between the beginning and ending inventory.

However, strata only measured on receipts and inventories may have a large systematic coefficient since the inventory difference for those would be quite large with the beginning and ending normally balancing nicely, but the receipts within that stratum do not have corresponding shipments—for example UF_6 received on values assigned by measurement systems B-1, S-1, E-1, and I-1 but the corresponding shipments are shipped as UO_2 in rods and assemblies using different bulk & sampling methods).

The systematic coefficients for a given stratum's measurement systems is given by

$$\left(C_{Sys_{BI}} + C_{Sys_A} - C_{Sys_R} - C_{Sys_{EI}}\right)^2$$

where

C_{sys} = the stratum grams U or ^{235}U as desired, and

BI, A, R, & EI are the material balance components

In simplest terms, if a certain material type (stratum) only appears on one MBC, say, additions, then the systematic coefficient is the square of the sum of SNM masses within that stratum.

4.1.6 Random Error Variances and Relative Standard Deviations (RSDs)

Random error variances are calculated using replicated measurements. This could be analyses of multiple samples from the same UO_2 powder blend or pellet lot or repeated weighings of unknown-weight items or control standard weights. The standard deviation (or, equivalently, the root mean square error from a one-way ANOVA) of paired differences or replicate ranges is used to estimate the random variance. For samples, some facilities use repeated measurements of replicate samples to perform a fully nested ANOVA to separate the sampling mean square error (reproducibility) from the analytical mean square error (repeatability) and propagate them independently from one another. Most facilities allow the sampling and analytical random error to remain confounded and propagate thusly.

The random variances may be pooled across like-kind measurement systems. For example, standard deviation of standard weighings across all pellet scales may constitute σ_{Rnd} for pellet tray weighing.

4.1.7 Random Sensitivity Coefficients

The listing of individual items, batches, or lots is key in calculating random sensitivity coefficients, which are simply the sum of squared SNM masses (grams U or grams ^{235}U as applicable) across all material balance components within the measurement system's stratum.

If only the stratum totals are available, then the random coefficient cannot be accurately determined unless each item within that stratum and material balance component is nearly identical in size. In this case, the square of the sum of SNM masses may be divided by the number of measurements made for that total. In this case, if these data are available, then the individual measurements could likely be queried as well. This will underestimate the total coefficient by the degree in which there are differences in the individual item sizes, which is almost always enough to produce a significant bias. This may be an adequate estimation method for a stratum with very small totals such as a certain type of waste drum or effluents to the environment without undue impact to the SEID and associated decision rules.

The random coefficients—once squared SNM masses for each measured value are obtained—are much simpler than the systematic coefficients. They are the sum of squared masses across all strata and material balance components applicable to each measurement system.

$$\sum_{\mathrm{MBC}}^{All\ MBCs} \sum_{\mathrm{STRATUM}}^{All\ Strata} \mathrm{C}^2_{\mathrm{Random}_{\mathrm{STRATA}_{\mathrm{MBC}}}}$$

where C^2 = the squared SNM gram (U or ^{235}U as appropriate) of each stratum

4.2 Individual ID Evaluation Toolbox

In USNRC regulated facilities, the entire material balance summary and evaluation are contained on the front of one page. The material balance components, ID, as well as all the values necessary for evaluating the ID and the capability of the SEID to detect losses are included in a simple chart. Additionally, most facilities use a graded approach for investigating lower-order IDs based on a the SEID times an expansion factor.

The key to this is the ID variance, which is the sum of random and systematic variances in the appropriate units (grams U or grams ^{235}U). The square root of this variance is the SEID. But the individual measurement and sampling systems each have a part or component of that variance associated with them in the same units. Therefore, these variances (recommend using the ^{235}U variances) may be summed across all uranium element sampling methods, for example, or all weighing systems, or across all measurement systems for an individual stratum (or all of them).

4.2.1 Graded Limits Based on SEID Coverage Factors

The lowest level of ID investigation at the facility that generated this document is twice SEID. If an inventory difference is less than twice the SEID, no further action is required. If the ^{235}U ID exceeds twice the SEID but is less than three times the SEID, the issue is entered into the facility's corrective action program for investigation. Similar actions are taken if the SEID exceeds 0.125% of the active inventory.

If the ID (U or ^{235}U) exceeds three times the SEID but is less than the detection threshold, a higher-level corrective action program investigation and formal causal analysis are initiated.

4.2.2 Active Inventory

The active inventory is simply the sum of all the stratum level systematic coefficient grams U or ^{235}U—not the stratum ID, but simply the sum of all the grams in all the strata after common terms are eliminated. This value is the best tool for evaluating the relative size of the SEID and changes to the SEID from year to year. In fact, if it is cost prohibitive to recalculate the SEID, the SEID may be made relative to the Active Inventory and applied to the next material balance period's active inventory to scale the SEID year-to-year as long as it can be demonstrated that there were not significant changes to key bulk, mixing, sampling, or analytical methods or the active inventory (say no more than ±25% active inventory change).

Using the systematic coefficient data by stratum, it's also straight forward to calculate the active inventory not only of the entire MBP, but of each measurement system. Each system type (bulk, sampling, analytical) should sum to 100% of the active inventory since all entries on the accountancy system are based on measurements. If a measurement system is used to measure at least 25% of the active inventory, then care should be taken in calibrations, bias evaluations, control limits, and measurement control event response protocols. A very small change in precision or drift could have a safeguards significant impact on the SEID or inventory difference itself.

4.2.3 Abrupt Loss Detection Limit Based on a Lower Limit of a Goal Quantity

For non-US facilities, this document recommends that stakeholders develop an abrupt loss (abrupt → one MBP) goal quantity that would indicate the need to shut down and re-inventory. This may be statistically driven, financially driven, security related, or some combination of all those things. Then, use the SEID to build a lower, one-sided limit based on an expansion factor such as 2 (> ~95% confidence), or 1.7 (≈ 90% confidence), or some similar criterion that all stakeholders agree is reasonable. This is called a detection threshold.

In the US, a violation of the detection threshold would be more indicative of a systemic accounting system failure—failure to include a key material type in the material balance equation, for example. This is more aligned with a major quality check of the accountancy system than a loss detection tool. Once-per-year material balance periods and single-MBA plants cannot isolate losses in time or space. Therefore, other tools should be used to find anomalies and losses in a more timely fashion even if sensitivity or specificity is sacrificed.

4.2.4 SEID Evaluation Against Active Inventory

The SEID may be made relative to the active inventory to provide stakeholders—especially external stakeholders such as corporate oversight or regulatory oversight—a way to standardize and interpret the overall measurement capability of your facility, especially when they have oversight of others. This is also an important component in historical ID analysis discussed below.

4.2.5 Key Measurement System Identification

Finally, another important role of the SEID model is to identify measurement systems and inventory strata that contribute the most to the uncertainty of the ID. It's the SEID model saying, "You have material unaccounted-for here!"

Two criteria are used in US facilities: one is whether the system is a cumulative 90% variance contributor to the ID when ranked by variance contribution, and the other is whether a system measures at least 25% of the active inventory.

The variance contributor analysis simply involves taking the list of all bulk, sampling, and analytical systems and rank ordering them by the magnitude of their ^{235}U variance. Then, at each row, take the sum of these variances from the top row to the current row divided by the total ID variance to get a cumulative percentage for each system similarly to a pareto analysis, only it analyzes variances instead of count/frequency data. The user will find that the percentage rounds to or exceeds 90% after a relatively few measurement systems are summed. Moreover, any system measuring at least 25% of the active inventory is automatically flagged regardless of the variance contribution.

These flagged "key" systems (not to be confused with KMPs, but truly "key" systems) should be ones used to measure feed materials received into the MBA and product materials being shipped out of the MBA. It may be that one or two are very large inventory strata that are neither shipments or receipts. However, if an NDA holdup technique, or some other inventory stratum seems to have a variance contribution percentage that far outweighs its active inventory percentage, these systems are the first in line for improvement. Improving these will improve detection capability, accounting accuracy, and fiscal responsibility for this valuable commodity. Conversely, if a >25% active inventory system's variance contribution percentage is much smaller than its active inventory percentage, this system is performing well overall.

4.3 Historical ID Evaluation Toolbox

A few things to note about historical ID evaluation and trending. The beginning of the trending period is very important. If a known and ongoing bias is being investigated, then the trend should be reset at the first sign that improvements have been made to verify. Otherwise, attempt to choose a beginning time that either marks a well-behaved ID or some system change likely to induce or fix a trend: key personnel changes, key measurement system updates, events involving losses or gains of substantial SNM quantities.

4.3.1 Sequential Probability Ratio Test (SPRT)

Chapter 14 of Reference [1] provides detailed calculation methodology for the sequential probability ratio test, which is a type of cumulative analysis and limits. Only, the ID data is transformed and the limits are based on log likelihood parameters associated with alpha error rate (false positive risk tolerance) and beta error rate (false negative risk tolerance). Therefore, this analysis contains the transformed cumulative data and two sideways V-shaped sets of limits expanding upward and downward as the sequence progresses. The space inside the inner V infers no trend. The space outside the outer V infers a statistical trend. The space between the V-shaped limits infers that more data is required to make a conclusion.

The strength of this analysis is that it provides some assurance of whether or not there is a trend as well as assurance that enough observations have been made to make such a conclusion—or not. The drawback of this analysis is that while the trend/no-trend results are easily demonstrated, the calculation method may be difficult to explain to management, regulators, or other stakeholders.

One simplification made for this document is that the current ID variance is assumed for the entire sequence to provide truly V-shaped limits. Individual ID variances provide more accurate decisions historically ("we used to have a trend, but now we do not," for example), but the direction of the V-shaped limits can change with each iteration. But, if this analysis is used as the definitive decision rule—yes, no, or needs more data—then it is good to have additional supporting analyses that show similar but more interpretable calculations.

4.3.2 Cumulative ID Analysis

Not included in Reference [1], but developed locally at the facility where this document was generated, the cumulative ID analysis is a fairly simple tool using the historical IDs and ID variances. These are cumulatively summed sequentially such that all IDs and variances are summed from the beginning of the trend period to the sequential year. The square root of the cumulative variances for each iteration may be multiplied by an expansion factor (±2 or ±3, for example) to provide upper and lower expanded limits that all stakeholders agree are appropriate for flagging bias investigations or accounting system changes. This method is easily interpreted as well—though there is no indication if not enough sequential data supports the conclusion. Both the calculation method and the result can be consumed by stakeholders of any background, technical or non-technical. Moreover, the expanding limits serve to scale the chart appropriately, not only serving as a definitive limit but also a visual aid to show the stakeholders how seriously to take a non-zero cumulative trend.

4.3.3 Sequential ID Analysis

The sequential ID analysis is simply the sequence of inventory differences over the trending period. Since the beginning inventory and ending inventory of contiguous periods are nearly identical, they are highly correlated, causing the sequence of IDs to display negative covariance. Thus, a healthy sequence of IDs performing as expected should have a general saw-tooth pattern that shows gains and losses around the expected value of zero. Since the cumulative analyses include error bars and limits, this one does not. The individual ID analysis provides the evaluations needed for each iteration.

4.3.4 Relative SEID Trending

The sequence of relative SEIDs (SEID ÷ Active Inventory) for each material balance period is plotted over the trending period. The slope of this line over time (more than three material balance periods) may be analyzed for statistical significance using any commonly-available statistical software package if desired to detect changes to the overall accountancy system capability over time. If a significant change—even an improvement (negative slope)—is detected, the cause should be associated with some change to one or more measurement or accounting systems. The figure shown shows a non-significant negative slope over time, indicating apparent continuous improvement, but not something that is necessarily statistically identifiable. Not also the sequential plot of the active inventory on the same chart for information.

5.0 COMPUTER CODES USED IN CALCULATION

The results reported in this document are not dependent on the software program used. This work may be independently reproduced using the methods and references described in this document.

6.0 CLOSING THOUGHTS

While this document does not provide detailed statistical formulations or an example SEID, the intended audience is management or regulators who are considering the implementation of SEID modeling to evaluate material balances— the possibilities, the costs, and the benefits.

There are ample opportunities to create decision rules that help quantify the overall health of a nuclear material accountancy system. There are a few potential areas of concern if implemented incorrectly, namely failure to include systematic uncertainties and their appropriately calculated sensitivity coefficients as well as failure to eliminate the common terms from the analysis to calculate the active inventory. Moreover, failure to sum the squared SNM masses of individually measured items, batches, or lots may also bias the random variance of the ID.

The question becomes whether a recently detected bias in measurements (that is, the inventory difference) or in the ID evaluation's decision rules (that is, the SEID), how much is tolerable? The rule of thumb used by the author of this document is that an ID contribution (potential bias on the accounting record) less than or equal to 10 grams ^{235}U or a ^{235}U SEID contribution less than or equal to 100 grams may be considered *de minimis*. Potential biases or ID impacts that exceed 500g ^{235}U are generally considered of safeguards significance. Between 10 and 500 grams ^{235}U, discretion may be used to minimize the aggregate impacts of small issues while balancing the cost-benefit of a graded approach to safeguards.

7.0 REFERENCES

[1] NUREG/CR-4604	Statistical Methods in Nuclear Material Accounting
[2] 10 CFR 74.31	US Title 10, Code of Federal Regulations, Part 74Material Control and Accounting for Special Nuclear Material of Low Strategic Significance