

# Evaluation of the Revised Sampling Planning Method for National Safeguards Inspection

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## Abstract

As the ROK adopted the state level approach (SLA) as the IAEA safeguards, the ROK's Nuclear Safety and Security Commission (NSSC) amended domestic notification to strengthen national safeguards inspection. The notification requires an independent on-site inspection supporting program to verify the facilities' declared information, which includes an optimized sampling method for on-site inspection.

This research suggests a revised sampling method for national inspection from the conventional IAEA's sampling planning method. The revised method minimizes assumptions for simplification in the conventional method to calculate the exact sample sizes for each verification type. The neglected assumptions in the conventional methods are the binomial assumption of the hypergeometric distribution and the parameter assumption using regression analysis. A previous study indicated that the revised method can reduce total sample size or sample size for more accurate verification types for each stratum, while maintaining the same non-detection probability ( $\beta$ ) given to a target facility.

This research then compares the feasibility of the revised method using a hypothetical physical inventory of a benchmark fuel fabrication plant and diversion scenarios to compare the detection capability of the two methods. The following four diversion scenarios are suggested and detection probabilities for each scenario are then compared to the non-detection probability given to the benchmark facility. The four diversion scenarios are 1) 1 SQ diversion in a single stratum with the gross defect, 2) 1 SQ diversion in a single stratum with the partial defect, 3) 1 SQ diversion in a single stratum with the bias defect, and 4) 1 SQ diversion in two strata with the gross defect.

Results indicate that the revised method satisfies the detection probability while maintaining the small sample sizes. Therefore, the revised sampling planning method can be applied to the on-site inspection supporting program for national inspection. Future works will include the feasibility of applying the independent sampling plan for national inspection to demonstrate the feasibility.

## Introduction

The International Atomic Energy Agency (IAEA) performs safeguards inspection to verify the declared information of member states' nuclear facilities. The safeguards inspection capability is only required by the agency under the comprehensive safeguards agreement (CSA). However, as the safeguards agreement between the agency and a member state changes to integrated safeguards (IS) and the state level approach (SLA), the agency draws safeguards conclusions on a member state considering their safeguards capability. Therefore, the importance of national safeguards inspection capability has increased. As a result, the Nuclear Safety and Security Commission (NSSC) of the Republic of Korea (ROK) recently made an amendment to a domestic notification on national safeguards inspection [1]. The amendment requires the national inspection to verify information regarding the mass, special nuclear material concentration and isotopic ratio of items declared by a facility. Therefore, the ROK requires independent sampling plan for national safeguards inspection.

A previous study examined the feasibility of applying the conventional IAEA sampling planning method on a national inspection and suggested a revised method, which neglects the assumptions made in the conventional method, to minimize approximations [2]. The results of this study indicated that the revised method can reduce "the total sample size" or "sample size of the precise verification method" compared to the conventional method while maintaining the same detection capability. Therefore, the revised method can reduce inspection burden.

However, the detection capability of the revised method for diversion scenarios must be evaluated. This study evaluates the consistency of the non-detection probability ( $\beta$ ) of the conventional method and the revised method using a benchmark fuel fabrication plant and four different diversion scenarios: 1) gross defect in a stratum, 2) partial defect in a stratum, 3) bias defect in a stratum, and 4) gross defect in two strata. The  $\beta$  between two sampling methods for four diversion scenarios were then compared using independent variable t-testing. When the  $\beta$  of the revised method is inconsistent with the conventional method, we then perform additional statistical testing to determine if the result of the revised method satisfies the  $\beta$  given to the facility. We then evaluate if the revised method is acceptable once the  $\beta$  of the revised method satisfies two statistical testing results.

## Revised Sampling Planning Method

The IAEA defines  $\beta$  as "the probability of non-detecting the defective items in a stratum after verifying samples taken from the stratum" [3]. The sample size in safeguards inspection is calculated as the minimum number of items in a stratum which satisfies the  $\beta$  given to the facility for one significant quantity (1 SQ) nuclear material diversion in a stratum. The  $\beta$  given to a facility depends on the sensitivity of the nuclear material in the facility and the safeguards agreement between the agency and member states (Table 1) [3]. For example, the  $\beta$  of a fuel fabrication plant (FFP) in the ROK using low enriched uranium (LEU) has an "integrated safeguards" agreement type and a "non-sensitive" facility type; therefore, the  $\beta$  of the facility becomes randomly low (0.8) [4].

Table 1. Different detection probabilities (1-β) and characteristics

Detection prob. (1-β)	Classification	Agreement Type	Facility type
0.9	High	CSA(+AP), IS	Sensitive
0.5	Medium	CSA(+AP)	Non-sensitive
0.2	Randomly low	IS	Non-sensitive

The IAEA establishes three main diversion scenarios (diversion into gross, partial, and bias defect) since an adversary can divert nuclear material using different diversion scenarios. The sample sizes for each diversion scenario are then calculated. The characteristics of the three diversion levels are described in Table 2 [3].

Table 2. Definitions of the three levels of defects

Defect Level	Verification Methods	Definition
Gross	Method H	Falsified item that all or most of the declared material is missing
Partial	Method F	Falsified item that some fraction of the declared material is missing
Bias	Method D	Falsified item that small fraction of the declared material is missing

The β of a stratum is calculated for diverting 1 SQ of nuclear material by each defect level using a hypergeometric distribution. The conventional method made approximations to calculate hypergeometric distributions in the initial sample sizes and maximum non-detection probability for all verification methods ( $Q_{max}$ ) since mobile systems in the early 1990s could not calculate the exact solution of a large number hypergeometric distribution. Therefore, the conventional method used binomial approximation and regression-based simplification for initial sample size calculation and sample size optimization [5]. However, due to recent significant advances in computing capability, these limitations are no longer an issue. Our previous study suggested a revised method to calculate the exact solution for β and  $Q_{max}$  [2]. Table 3 describes the differences between the conventional and revised sampling methods.  $D$  indicates the number of items with gross defect.  $Q_{max}$  and  $\beta_{HFD}$  indicate the maximum non-detection probability of a stratum for all possible defect scenarios and the non-detection probability of a stratum for a specific defect scenario using method H, F and D.  $m_{00}$  indicates the maximum number of defected items which can be detected using the most precise method.  $w_1$  and  $w_2$  indicate the number of defected items among samples for the gross and partial defect verification, respectively. Our previous study shows that the revised method has a smaller total sample size or a sample size of a more precise verification method, while maintaining the same detection probability [2].

Table 3. Differences between the conventional and revised methods

	Conventional method	Revised method
Initial sample size (Gross)	$\beta \approx \left(1 - \frac{n}{N}\right)^D$	$\beta = \frac{\binom{D}{0}\binom{N-D}{n}}{\binom{N}{n}}$

Initial sample size (Partial, Bias)	$\beta \approx \left(1 - \frac{m(\gamma)p}{N}\right)^n$	$\beta = \sum_{j=0}^n \frac{\binom{m(\gamma)}{j} \binom{N-m(\gamma)}{n-j}}{\binom{N}{n}} (1-p)^j$
	$Q_{max} = \beta_H \beta_F \beta_D$	$Q_{max} = \max(\beta_{HFD,m}) \quad (D \leq m \leq m_{00})$
Max. non-det. prob. ( $Q_{max}$ )	$\beta(H \text{ or } F \text{ or } D) \approx \begin{cases} \left(1 - \frac{m(\gamma)p}{N-0.5(n-1)}\right)^n & (n \leq m(\gamma)p) \\ \left(1 - \frac{n}{N-0.5(m(\gamma)p-1)}\right)^{m(\gamma)q} & (n > m(\gamma)p) \end{cases}$	$\beta_{HFD} = \sum_{j=0}^n \frac{\binom{m}{j} \binom{N-m}{n-j}}{\binom{N}{n}} \sum_{w_1=1}^{W_1} \sum_{w_2=1}^{W_2} \frac{\binom{n_H}{w_1} \binom{n_F}{w_2} \binom{n_D}{j-w_1-w_2}}{\binom{n}{j}} (1-p_H)^{w_1} (1-p_F)^{w_2} (1-p_D)^{j-w_1-w_2}$

## Evaluation Methods

For this study, we designed a benchmark fuel fabrication plant and four diversion scenarios to evaluate the detection capability of the revised method. We compared the  $\beta$  between the conventional and revised sampling methods once an adversary diverts 1 SQ of nuclear material in the benchmark facility using the four scenarios.

### Benchmark Facility Configuration

The benchmark facility imports UF<sub>6</sub> cylinders, consists of the reconversion and sintering process, and exports UO<sub>2</sub> pellets (Figure 1). All items are classified into three strata: 1) UF<sub>6</sub> cylinders (UF), 2) UO<sub>2</sub> powder (PD), and 3) UO<sub>2</sub> pellets (PL). The UF stratum is verified using method H and F and the PD and PL strata are verified using method H, F and D. The characteristics and relative uncertainty of each verification method for all strata are described in Table 4. The target  $\beta$  is 0.8, as described in Table 1.

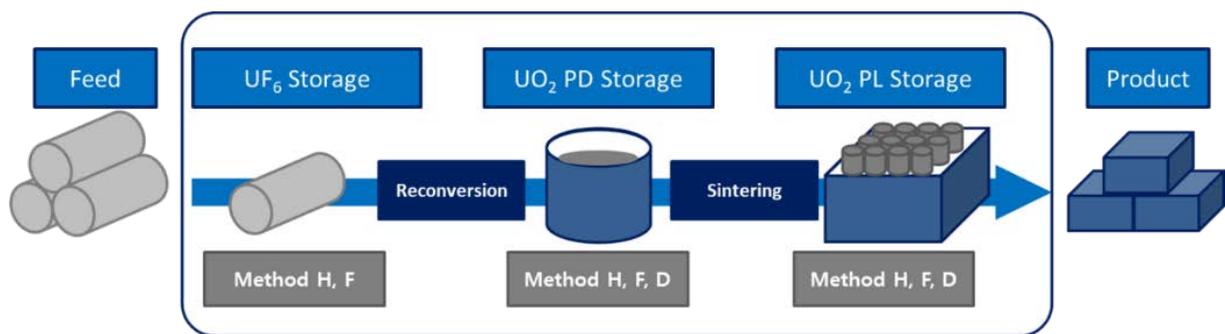


Fig. 1. Configuration of a benchmark fuel fabrication plant.

Table 4. Characteristics of verification methods for three strata

Stratum	Method H		Method F		Method D	
	Instrument	$\delta$ (RSD)	Instrument	$\delta$ (RSD)	Instrument	$\delta$ (RSD)
UF	HM-5	0.1500	IMCN	0.0488	-	-

PD	HM-5	0.1500	IMCN	0.0542	GRAV/TIMS	0.0070
PL	HM-5	0.1500	IMCN	0.0362	GRAV/TIMS	0.0051

The list of inventory items (LII) of the benchmark facility was the modified LII of a fuel fabrication plant in the ROK. This study calculated the  $\beta$  of a non-homogeneous LII to demonstrate an on-site inspection. The characteristics of several items of the benchmark LII is depicted in Figure 2. However, the characteristics (net weight, U concentration and  $^{235}\text{U}$  enrichment) of each item are not homogeneous. Table 5 shows the summarized results of each stratum and characteristics of their corresponding verification systems.

Fig. 2. Characteristics of  $\text{UF}_6$  cylinders in the benchmark LII.

KMP	MDC	Qty	GrossWeight	NetWeight	UConcentration	UWeight	Enrichment	U235Weight
A	OGRB	1	2869.600	2230.600	67.610	1508.109	4.646	70.064
A	OGRB	1	2856.400	2233.000	67.610	1509.731	4.646	70.139
A	OGRB	1	2860.800	2233.400	67.610	1510.002	4.649	70.205
A	OGRB	1	2754.400	2116.800	67.610	1431.168	4.647	66.501
A	OGRB	1	2752.000	2114.000	67.610	1429.275	4.647	66.413
A	OGRB	1	2754.800	2113.300	67.610	1428.802	4.647	66.391
A	OGRB	1	2557.800	1937.000	67.610	1309.606	4.658	60.999
A	OGRB	1	2883.000	2267.200	67.610	1532.854	4.652	71.308
A	OGRB	1	2548.600	1931.800	67.610	1306.090	4.646	60.676
A	OGRB	1	2564.200	1940.000	67.610	1311.634	4.658	61.095
A	OGRB	1	2882.800	2253.200	67.610	1523.389	4.651	70.854
A	OGRB	1	2862.400	2230.500	67.605	1507.935	2.202	33.205
A	OGRB	1	2842.300	2201.600	67.612	1488.546	4.650	69.217
A	OGRB	1	2875.500	2243.300	67.612	1516.740	4.652	70.559

Table 5. Summarized LII of the benchmark facility

Stratum	N	M (kg)	x (kg)	Method H ( $\delta_H$ )	Method F ( $\delta_F$ )	Method D ( $\delta_D$ )
UF	306	18,903	61.8	0.1500	0.0488	-
PD	139	1,842	13.3	0.1500	0.0542	0.0070
PL	2,066	3,444	1.67	0.1500	0.0362	0.0051

The diversion scenarios were 1 SQ diversion using the 1) gross defect in a stratum, 2) partial defect in a stratum, 3) bias defect in a stratum, and 4) gross defect in two strata (0.5 SQ diversion in UF-PD, UF-PL and PD-PL stratum). We selected the defect size for partial and bias defects as two times that of the relative uncertainty of the ‘IMCN’ (30 %) and ‘GRAV/TIMS’ (10 %), respectively.

### *Non-detection probability ( $\beta$ ) evaluation*

The  $\beta$  of the revised method was evaluated using a three-step approach. In Step 1, the  $\beta$  distribution of all strata for all diversion scenarios is calculated using the revised and conventional methods. In Step 2, the consistency of the  $\beta$  distributions between the conventional and revised method are tested. If the two  $\beta$  distributions are consistent, we evaluate the  $\beta$  of the revised method is acceptable. If not, the  $\beta$  of the revised method is evaluated in the next step. In step 3, the  $\beta$  of

the revised method is tested to determine if it is smaller than the  $\beta$  of the facility (0.8). If the  $\beta$  of the revised method is smaller than 0.8, we evaluate it is acceptable. Figure 3 depicts the overall  $\beta$  evaluation process of the revised method. The detailed processes for each step are described below.

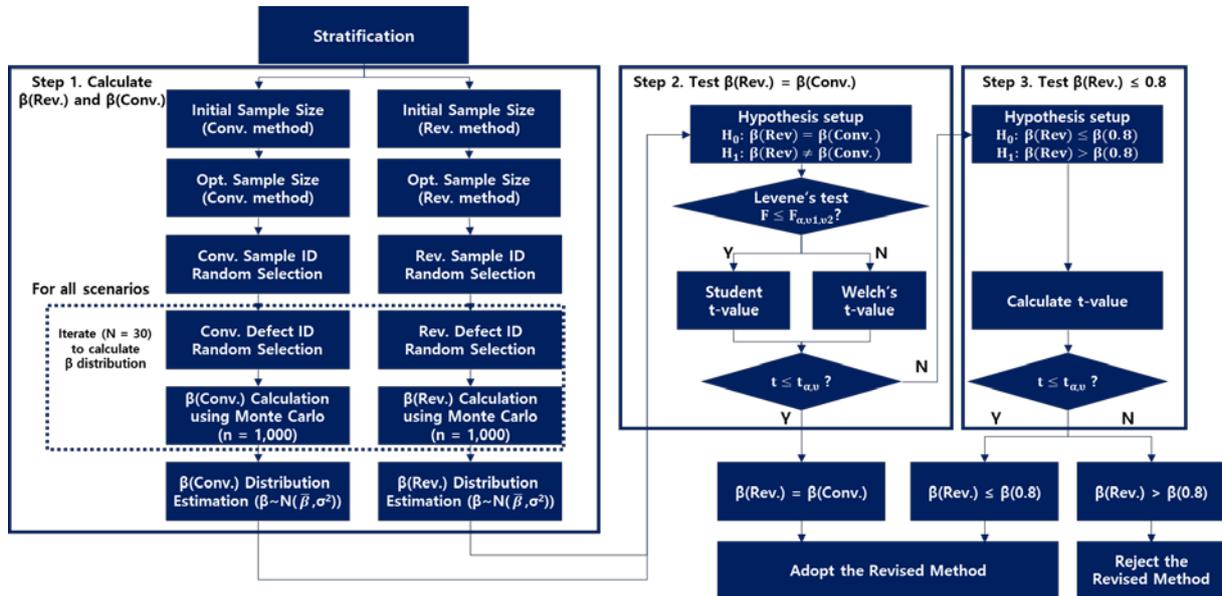


Fig. 3. Flowchart to evaluate the non-detection probability ( $\beta$ ) of the revised method.

We calculated the  $\beta$  of the two methods for all diversion scenarios, which consists of the following procedures.

- (1) Calculate initial sample sizes for each stratum using the conventional and revised methods.
- (2) Calculate optimized sample sizes for each stratum using both methods.
- (3) Select the sample ID of both methods for each stratum using random selection.
- (4) Select the defect ID of both methods for each scenario and stratum using random selection.
- (5) Calculate  $\beta$  of both methods for the scenario using a Monte Carlo simulation.
- (6) Calculate  $\beta$  of both methods for all diversion scenarios by repeating procedures (4) and (5).
- (7) Estimate the  $\beta$  distribution of both methods for all diversion scenarios by iterating procedures (4), (5) and (6) 30 times.

The Monte Carlo simulation was applied to calculate the non-detection probability in procedure (5). We compared the ID of the sample and defect for each stratum and diversion scenario. Once the sample and defect ID were consistent, we calculated the detection probability with approximately a 95 % confidence interval ( $k=2$ ) for the item using the defect size and relative uncertainty of the verification method. We then simulated whether the verification method can detect the defect. A random number between 0 and 1 was generated to simulate the measurement result of the verification system. The random number was then compared with the calculated detection probability of the item. If the random number was smaller than the calculated detection probability, we determined that the item is not detected. The stratum was considered non-detected for the diversion scenario if all items with a consistent ID were not detected. We calculated the  $\beta$

for each stratum and diversion scenario by repeating the evaluation process 1,000 times. The  $\beta$  for the two strata diversion scenario was evaluated by multiplying the  $\beta$  of the two strata with 0.5 SQ nuclear material diversion. We then iterated the overall process 30 times to estimate the  $\beta$  distribution.

We then evaluated whether the two  $\beta$  distributions are consistent using an independent variable t-test. The null hypothesis for the test was, “the  $\beta$  distribution of the two methods is consistent”. We assumed that both  $\beta$  have normality since the number of iterations to estimate the distribution was 30 [6]. We then conducted the Levene’s test (F-test) to evaluate the consistency of variances between the two distributions [7]. The Student’s and Welch’s t-value were calculated for the cases with homogeneous and heterogeneous variance, respectively [8, 9]. The calculated t-value was then compared to the critical value ( $t_{\alpha/2, v}$ ). If the  $\beta$  of the revised method was inconsistent with the conventional method, we then evaluated whether the  $\beta$  of the revised method is smaller than 0.8, which is the  $\beta$  of the benchmark facility. The null hypothesis became, “the  $\beta$  of the revised method is smaller than 0.8”. We then calculated the t-value for all cases and compared it to the critical value ( $t_{\alpha, v}$ ). We determined that the revised method is acceptable if the  $\beta$  were consistent to the conventional method or smaller than 0.8.

## Evaluation Results

The evaluation results for the  $\beta$  of the revised method using the benchmark facility and the evaluation method described above are summarized below. The calculated optimized sample sizes for both sampling methods are depicted in Table 6. These results indicate the revised method has a smaller total sample size (PD stratum) or smaller sample size with the precise verification method (UF stratum), which reduces the inspection burden. We then calculated the parameters (average and standard deviation) of the  $\beta$  distributions ( $\beta \sim N(\bar{\beta}, \sigma(\beta))$ ) for all diversion scenarios by following step 1. The results of Table 7 indicate that the benchmark facility is vulnerable for the diversion with gross defects since the number of samples is much smaller than the number of items for all strata.

We then evaluated if the  $\beta$  of the revised method is consistent with the conventional method or smaller than 0.8 by following steps 2 and 3. Table 8 includes the evaluation results for the 12 cases. These results indicate that the  $\beta$  of the revised method is consistent for six cases and the  $\beta$  is smaller than 0.8 for the other cases. They also indicate that the  $\beta$  inconsistency usually occurred for the PD stratum where the total sample size reduced. Therefore, the results of the benchmark study show that the revised method can reduce the inspection burden compared to the conventional method while maintaining the  $\beta$ .

Table 6. Optimized sample size results using both sampling methods

Stratum	Conventional (IAEA) Sampling Method			Revised Sampling Method		
	Gross	Partial	Bias	Gross	Partial	Bias
UF	19	14	0	21	12	0
PD	4	1	1	3	1	1
PL	8	2	1	8	2	1

Table 7. Non-detection probability distribution using both methods for all diversion scenarios

Conventional (IAEA) Sampling Method												
	Gross Defect			Partial Defect			Bias Defect			Two Strata Defect		
	UF	PD	PL	UF	PD	PL	UF	PD	PL	UFPD	UFPL	PDPL
$\bar{\beta}$	0.796	0.767	0.779	0.568	0.425	0.450	0.285	0.031	0.085	0.772	0.785	0.772
$\sigma(\beta)$	0.014	0.016	0.014	0.013	0.015	0.013	0.014	0.006	0.010	0.013	0.014	0.010
Revised Sampling Method												
	Gross Defect			Partial Defect			Bias Defect			Two Strata Defect		
	UF	PD	PL	UF	PD	PL	UF	PD	PL	UFPD	UFPL	PDPL
$\bar{\beta}$	0.797	0.784	0.773	0.571	0.462	0.438	0.298	0.037	0.089	0.781	0.779	0.771
$\sigma(\beta)$	0.012	0.011	0.012	0.017	0.021	0.013	0.013	0.006	0.010	0.013	0.013	0.014

Table 8. Evaluation of non-detection probability of the revised and conventional sampling methods

Step 2) H0: $\beta(\text{Rev.}) = \beta(\text{Conv.})$ , H1: $\beta(\text{Rev.}) \neq \beta(\text{Conv.})$ , $\alpha(0.05)$ , $n1 = 30$ , $n2 = 30$												
Test	Gross Defect			Partial Defect			Bias Defect			Two Strata Defect		
Results	UF	PD	PL	UF	PD	PL	UF	PD	PL	UFPD	UFPL	PDPL
F-test <sup>1)</sup>	S	W	S	S	W	S	S	S	S	S	S	W
t-test	Yes	No	Yes	Yes	No	No	No	No	Yes	No	Yes	Yes
Step 3) H0: $\beta(\text{Rev.}) \leq \beta(0.8)$ , H1: $\beta(\text{Rev.}) > \beta(0.8)$ , $\alpha(0.05)$ , $n = 30$												
Test	Gross Defect			Partial Defect			Bias Defect			Two Strata Defect		
Results	UF	PD	PL	UF	PD	PL	UF	PD	PL	UFPD	UFPL	PDPL
t-test <sup>2)</sup>	-	Yes	-	-	Yes	Yes	Yes	Yes	-	Yes	-	-

1) S: Student t-value, W: Welch's t-value.

2) Step 3 is performed for strata with  $\beta$  inconsistency.

## Discussion

The results of the  $\beta$  evaluation indicate that the revised sampling method can reduce inspection burden while maintaining the required  $\beta$  of the facility. The inspection burden reduction will be more significant if the sensitivity of the target facility increases.

The material accounting and evaluation methods of the IAEA, including on-site sampling plan, was established from the 1970s to the early 1990s. Therefore, the accounting system of the agency inevitably contains approximations due to the computational limitations of that time. As we examined in this study, eliminating the approximation results in a slight reduction of inspection burden. Once the approximations in the overall accounting systems of the agency are revised, a significant inspection burden reduction will be achieved in the future.

## Conclusion

Independent sampling plan in national safeguards inspection has grown in importance along with that of national safeguards inspections in the ROK. As a result, a revised sampling planning method for national safeguards inspection was developed. This revised method removed the approximations to simplify calculations in the conventional method. This study evaluated the  $\beta$  of

the revised sampling method compared to the conventional sampling method. We compared the  $\beta$  of the revised method using a benchmark fuel fabrication plant and the following steps: 1) the  $\beta$  distribution was calculated for four diversion scenarios (gross defect, partial defect, bias defect in a stratum, and gross defect in two strata) using the conventional and revised method, 2) the consistency between the  $\beta$  distributions of the conventional and revised methods was analyzed, and 3) if the  $\beta$  distribution of the revised method was smaller than that the benchmark facility (0.8) was evaluated.

Our results indicate that the revised method reduces the total sample size or sample sizes of the sensitive verification method of a stratum compared to the conventional method while maintaining an identical  $\beta$  or having a  $\beta$  smaller than 0.8. Therefore, the revised method reduces the inspection burden by removing approximations made in the conventional method. Since the IAEA made a number of approximations in nuclear material accounting in the past, the overall approximation removal may result in significant resource optimization of national safeguards inspection in the ROK. Future works will seek to identify the approximations in the IAEA accounting system and evaluate the effect of these approximations.

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