Random Inspection Planning for Misuse Detection in Safeguards

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Abstract

The IAEA uses random inspections (RIs) to, inter alia, provide credible assurance that declared nuclear facilities are not used for undeclared purposes. These inspections are random in the sense that they are scheduled randomly in date and time, with short notice given to the inspected site. The IAEA has interest in employing statistical models for RI planning that take advantage of any potential efficiency gains while maintaining a high level of effectiveness.

This paper first introduces the model parameters that are necessary for a quantitative analysis of RI models for misuse inspections (subsequently referred to as RI models) and discusses their importance. Then, using the model parameters, the set of all RI models is introduced, and three example RI models are presented. Next, for any RI model the probability is derived that any facility is selected at least once per year for an RI, and – regarding the objective of an RI – the probability that a misuse is detected within T days after its start, where the parameter T is the duration of misuse signatures at the facility. Next, the question is addressed which RI model should be chosen for RI planning: If no further constraints from the IAEA are imposed on the RI models (e.g., maximum unpredictability of the number of RIs in each year, resource constraints leading to an upper number of RIs, etc.), then the RI model that maximizes the achieved detection probability for a given set of input parameters should be selected. This maximization problem, however, is by no means trivial, because the maximization is performed over a set of RI models and not over a subset of real numbers.

Finally, the functionality and features of the software prototype TRIPS (Tool for Random Inspection Planning in Safeguards) are demonstrated, and future work topics are highlighted.

Keywords: Random Inspection Model, Detection Probability, Indicator Time Signature, Tool for Random Inspection Planning in Safeguards (TRIPS)

1 Introduction and Motivation

The IAEA uses random inspections (RIs) to, inter alia, provide credible assurance that declared nuclear facilities are not used for undeclared purposes. An effective and efficient planning regime for scheduling RIs and evaluation of achieved selection (and potentially detection) probabilities represents an important building block in developing State-level safeguards approaches (SLA) under the State-level Concept (SLC). These inspections are random in the sense that they are scheduled randomly in date and time, with short notice given to the inspected site; see [1]. While RIs can also be performed to detect the

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diversion from declared material, the focus in this paper lies on the detection of misuse in declared facilities. Examples include undeclared reprocessing of spent fuel in a declared reprocessing plant, undeclared enrichment in a declared enrichment facility, irradiation of targets in a research reactor, fabrication of targets in a fabrication plant, or undeclared feed in a conversion plant.

An RI model for misuse inspections is a process for sampling random inspection dates. For brevity this paper will use the term *RI model* always implying misuse inspections. The aim of this paper is to describe the basic framework for RI models and to formalize, with demonstration, the objectives of RIs. The formal objectives can be used to identify RI models that take advantage of any potential efficiency gains while maintaining a high level of effectiveness.

The paper is organized as follows: Section 2 introduces model/input parameters necessary to perform a quantitative analysis of RI models and discusses three RI models. In Section 3, the objectives of RIs are formalized and the general equations of the selection as well as the detection probability are determined. Furthermore, an example scenario is discussed followed by a discussion on RI model selection. Section 4 briefly introduces the Tool for Random Inspection Planning in Safeguards (TRIPS). Conclusions and future work are highlighted in Section 5.

2 Random inspection models

This section introduces 1) the initial situation and the model parameters necessary to perform a quantitative analysis of RI models, 2) the set of RI models, and 3) presents three examples of RI models with a discussion of their respective (dis)advantages.

2.1 Model/input parameters and the set of RI models

The initial situation is as follows: A population of similar facilities in a state is safeguarded against misuse using an RI scheme. If one of these facilities were misused, detectable evidence of this misuse is only present for a finite duration T. In other words, this situation models transient signatures; persistent signatures are not considered here.

For the RI model, a reference time interval (e.g., one year) is considered that is subdivided into N days $1, \dots, N$ at which an RI can be performed; see Figure 1. Also, it is assumed that an RI can be scheduled on any day, i.e., there are no gaps in the timeline and there might be RIs also at weekends and national holidays. This assumption is essential for the derivation of the RI detection probability.

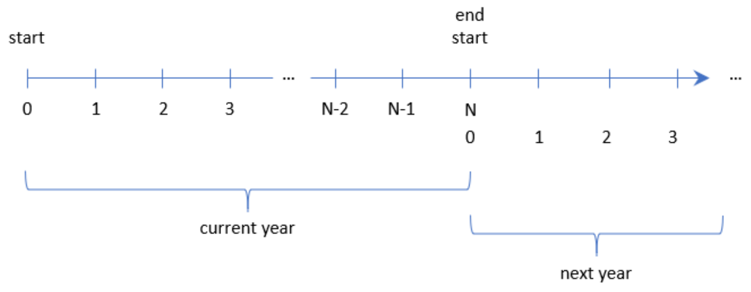

Figure 1: Timeline.

The model/input parameters for the RI models are as follows (see [2] and [3]): Let N be the number of discrete opportunities at which an RI can be performed during the reference period, usually 365 (days); let n be number of facilities in the State to which the RI scheme is applied; let k, $k = 0,1, \dots, N-1$, be the minimum number of RIs to be performed; let μ, $k < \mu < N$, be the expected number of RIs (μ can be a rational number); let p_{id} be the probability that a single inspection identifies misuse, given that it occurs at the misusing facility while indicators exist; and let T, $T < N$, be the indicators' duration, i.e., the number of discrete opportunities at which the misuse can be detected (measured in the same unit as N).

Note that a reliable estimate of p_{id} and T must be provided by experts. The software prototype discussed in Section 4 includes functionality to estimate the model sensitivity to these parameters.

2.2 The set of RI models

To formalize the way inspection days are chosen, we introduce the set X of RI models. Let the tuple $(i_1, i_2, \ldots, i_N) \in \{0,1\}^N$ denote the inspections days: $i_j = 1$ indicates that at day j an RI is performed and $i_j = 0$ indicates that at day j no RI is performed. If $p_{(i_1,i_2,\ldots,i_N)}$ denotes the probability that the inspection days $(i_1, i_2, \ldots, i_N)$ are chosen, then the set X of RI models is

$$X \coloneqq \left\{ \boldsymbol{p} \in [0,1]^{2^N} : p_{(i_1,i_2,\ldots,i_N)} = 0 \text{ for all } (i_1, i_2, \ldots, i_N) \in \{0,1\}^N \text{ with } \sum_{j=1}^{N} i_j < k \right. \\ \left. \sum_{(i_1,i_2,\ldots,i_N)\in\{0,1\}^N} p_{(i_1,i_2,\ldots,i_N)} = 1 \quad \text{and} \quad \sum_{(i_1,i_2,\ldots,i_N)\in\{0,1\}^N} p_{(i_1,i_2,\ldots,i_N)} \sum_{j=1}^{N} i_j = \mu \right\}. \tag{1}$$

Three comments on Eq. (1): The sum in the first line is because at least k RIs must be performed. The first equation in the second line is because $p_{(i_1,i_2,\ldots,i_N)}$ are probabilities, and the second equation in the second line formalizes the requirement that the expected number of RIs is μ. Note that set X contains RI models with the property $\mathbb{P}(\text{number of RIs} = k) = 0$; see RI model 1.

The set X contains uncountably many RI models, three of them are presented in the following for illustration. Note that they are not meant to reflect a particular choice of model by the IAEA. Also note that we leave it to the reader to show that the RI models specified in Eqs. (2)–(4) indeed belong to the set X.

Example: RI model 1

An RI model with a high predictability of the number of RIs concentrates this number around μ, so that only tuples $(i_1, i_2, \ldots, i_N) \in \{0,1\}^N$ with exactly[2] $\lfloor \mu \rfloor$ or exactly $\lfloor \mu \rfloor + 1$ RIs (indicated by elements equal to "1") have a nonzero probability:

$$p_{(i_1,i_2,\ldots,i_N)} = \begin{cases} \dfrac{1 + \lfloor \mu \rfloor - \mu}{\binom{N}{\lfloor \mu \rfloor}} & \text{for} \quad \displaystyle\sum_{j=1}^{N} i_j = \lfloor \mu \rfloor \\ \dfrac{\mu - \lfloor \mu \rfloor}{\binom{N}{\lfloor \mu \rfloor + 1}} & \text{for} \quad \displaystyle\sum_{j=1}^{N} i_j = \lfloor \mu \rfloor + 1 \\ 0 & \text{else} \end{cases} . \tag{2}$$

In this RI model resource planning is easy because only $\lfloor \mu \rfloor$ or $\lfloor \mu \rfloor + 1$ RIs are performed. If μ is an integer, then $\mu = \lfloor \mu \rfloor$ and Eq. (2) reduces to a uniform distribution with equal probability $\binom{N}{\mu}^{-1}$ over the set of all $(i_1, i_2, \ldots, i_N) \in \{0,1\}^N$ with exactly μ RIs. Note Eq. (2) is independent of k. Thus, all RI models with a minimum number of $0, 1, \ldots, \lfloor \mu \rfloor - 1$ RIs have the same probabilities as in Eq. (2). Also note that $\mathbb{P}(\text{number of RIs} = k) = 0$ for $k < \lfloor \mu \rfloor$.

Example: RI model 2

The inspection days in this RI model are generated as follows:

1. For any day d, $d = 1, \ldots, N$, an RI is scheduled with probability p ($p \in (0,1]$) independently of the history, i.e., independently of the RIs scheduled for days $1, \ldots, d-1$ (for $d > 1$). No RI is scheduled for day d with probability $1 - p$ (Bernoulli trial, coin toss).

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[2] The floor function $\lfloor \mu \rfloor$ maps μ to the greatest integer less than or equal to μ.

2. If the total number of RIs is larger or equal to k, then use the inspection days generated in Step 1. Otherwise, start again with Step 1. If $k = 0$, then Step 2 is omitted.

Using the binomial distribution (see [4] or [5]), the probability $p_{(i_1,i_2,\dots,i_N)}$ that the inspection days $(i_1, i_2, \dots, i_N)$ are chosen is given by

$$p_{(i_1,i_2,\dots,i_N)} = \begin{cases} \dfrac{p^\ell (1-p)^{N-\ell}}{1 - \sum_{j=0}^{k-1} \binom{N}{j} p^j (1-p)^{N-j}} & \text{for} \quad \ell = \sum_{j=1}^{N} i_j \geq k \\ 0 & \text{for} \quad \sum_{j=1}^{N} i_j < k \end{cases} , \tag{3}$$

where p is determined such that the expected number of RIs is μ, i.e.,

$$\frac{1}{1 - \sum_{j=0}^{k-1} \binom{N}{j} \, p^j (1-p)^{N-j}} \sum_{\ell=k}^{N} \ell \binom{N}{\ell} p^\ell (1-p)^{N-\ell} = \mu \, .$$

In the first line of Eq. (3), the denominator is the probability of sampling at least k RIs in Step 1, and the numerator is the probability to get the tuple $(i_1, i_2, \dots, i_N)$ with exactly $\ell = \sum_{j=1}^{N} i_j$ RIs in Step 1. Thus, Eq. (3) is the conditional probability that the inspection days $(i_1, i_2, \dots, i_N)$ are chosen under the condition that at least k RIs are performed.

In contrast to RI model 1, here any number of RIs between k (i.e., $\mathbb{P}(\text{number of RIs} = k) > 0$) and N could be sampled, which makes resource planning more difficult.

Example: RI model 3

The inspection days in this RI model are generated as follows:

1. Choose exactly k of the N days to hold the minimum number of RIs by sampling without replacement.

2. Choose additional RI days using a Bernoulli trial (coin toss) with probability q to perform an RI for each of the remaining $N - k$ days.

If $k = 0$, then the RI models 2 and 3 are equivalent with $p = q$. When $k > 0$, Steps 1 and 2 yield $k + (N - k)q$ expected RIs, and thus $q = (\mu - k)/(N - k)$. Note that $p \neq q$ for $k \geq 1$.

The probability $p_{(i_1,i_2,\dots,i_N)}$ that the inspection days $(i_1, i_2, \dots, i_N)$ are chosen is given by

$$p_{(i_1,i_2,\dots,i_N)} = \begin{cases} \dfrac{\binom{N-k}{\ell-k} q^{\ell-k} (1-q)^{N-\ell}}{\binom{N}{\ell}} & \text{for} \quad \ell = \sum_{j=1}^{N} i_j \geq k \\ 0 & \text{for} \quad \sum_{j=1}^{N} i_j < k \end{cases} . \tag{4}$$

Equation (4) is justified as follows: The number of RIs sampled in Step 2, $\ell - k$, follows a binomial distribution of $N - k$ trials, each with probability q of success. Therefore, the probability of sampling exactly ℓ RIs in Steps 1 and 2 (combined) is the numerator of Eq. (4). Among the $\binom{N}{\ell}$ tuples that sum to exactly ℓ RIs, all tuples have equal probability. The days are exchangeable, so the tuples are exchangeable. Therefore, the denominator is needed when referring to a specific tuple. For example, suppose $k = 2$, $N = 5$, and $q = 0.5$. The probability of sampling $\ell = 3$ RIs is $\binom{N-k}{\ell-k} q^{\ell-k}(1-q)^{N-\ell} = 37.5\%$, but the probability of sampling exactly (1,1,0,0,1) is 3.75% (because there are $\binom{N}{\ell} = 10$ ways to sample $\ell = 3$).

As in RI model 2, any number of RIs between k (i.e., $\mathbb{P}(\text{number of RIs} = k) > 0$) and N occurs with positive probability that again makes resource planning difficult. In the limiting case $k = \mu$ (note that $k < \mu$ is assumed in this paper) all three RI models coincide with p approaching zero and $q = 0$; see [3].

3 Formalization of RI objectives

To develop new or analyze existing RI models, the objective "misuse detection" must be quantified. That is done here in terms of the detection probability (DP). Using the signature time T, the probability of detecting the misuse is defined as the probability that a misuse is detected within T days after its start.

In the context of RI models, it is also of interest to determine the selection probability (SP) defined here as the probability that a particular facility (out of the pool of facilities to which the RI scheme is applied) is selected at least once per year for a RI.

Because misuse considerations are not needed for the SP derivation, we first derive the SP in Section 3.1, and subsequently the DP in Section 3.2.

3.1 Selection probability

Consider for any facility j the event $A_j = \{\text{facility } j \text{ is selected at least once per year for a RI}\}$, where $j = 1, \ldots, n$. Then the SP is the probability of the event A_j, and is given by, see [3],

$$\mathbb{P}(A_j) = 1 - \sum_{\ell=k}^{N} \mathbb{P}\left(\begin{Bmatrix} \ell \text{ RIs are performed} \\ \text{during the year} \end{Bmatrix}\right)\left(1 - \frac{1}{n}\right)^{\ell}, \tag{5}$$

where the events $\{\text{not selecting facility } j \text{ for the first RI}\}, \ldots, \{\text{not selecting facility } j \text{ for the } \ell-\text{th RI}\}$ are assumed to be statistically independent, and equally distributed on $\{1, \ldots, n\}$ for each single event.

Because the probability $\mathbb{P}(\ell \text{ RIs are performed during the year})$ depends on the RI model, it is different for all of the RI models in Section 2.2. The SPs for RI models 2 and 3 coincide by definition for $k = 0$, and there is a strong numerical evidence that the SP of RI model 1 is greater than the SP of RI model 3 which is greater or equal than the SP of RI model 1 for all pairs (k, μ) with $k < \mu$; see [3].

3.2 Detection probability

To derive the DP, the following misuse assumptions are made:

- The misuse is started in exactly one of the n facilities at any day $d = 0, \ldots, N-1$ (worst case from the detection view, without loss of generality: facility 1);
- If the misuse is started in facility 1 at the same day at which an RI is performed in facility 1, then the misuse is not detected that day;
- The misuse can only be detected within T days after its start (the detection window consists of T days).

If the misuse is started on day d, then it can only be detected on days $d+1, \ldots, d+T$. If no RI is performed on days $d+1, \ldots, d+T$ in facility 1, but, e.g., an RI on day $d+T+1$ in facility 1, then it is too late for detection. Two cases must be distinguished: if the misuse is started on days $d = 0,1, \ldots, N-T$, then the detection window lies in the current year (if $d = N-T$ then it can be detected at days $N-T+1, \ldots, N$ which are T days); or, if the misuse is started on days $d = N-T+1, \ldots, N-1$, then the detection window overlaps with both the current and subsequent year.

Let T' with $1 \leq T' \leq T$. If the misuse is started on day d, $d = 0, \dots, N - T'$, then the detection window is within the current year, and the probability that the misuse is not detected within T' days after its start on day d, is, see [3],

$$\mathbb{P}_{T'}^{(d)}(\text{not detecting the misuse}) = \sum_{\ell=Max(0,k-(N-T'))}^{T'} \mathbb{P}\begin{pmatrix} \ell \text{ RIs are performed during} \\ \text{days } d+1, \dots, d+T' \end{pmatrix} \left(1 - \frac{p_{id}}{n}\right)^{\ell}. \quad (6)$$

Again, the probability $\mathbb{P}(\ell \text{ RIs are performed during days } d+1, \dots, d+T')$ depends on the RI model. Using Eq. (6), the probability $DP(d)$ that a misuse is detected within T days after its start, is given by

$$DP(d) = \begin{cases} 1 - \mathbb{P}_{T}^{(d)}(\text{not detecting the misuse}) & : \quad d = 0, \dots, N - T \\ 1 - \mathbb{P}_{N-d}^{(d)}(\text{not detecting the misuse}) & : \quad d = N - T + 1, \dots, N - 1. \\ \qquad \times \mathbb{P}_{T-(N-d)}^{(0)}(\text{not detecting the misuse}) & \end{cases} \quad (7)$$

The reason for multiplying the non-DPs in the case $d = N - T + 1, \dots, N - 1$ is that the event $\{\text{not detecting the misuse at days } d, \dots, N\}$ happening in the current year is assumed to be statistically independent from the event $\{\text{not detecting the misuse at days } 1, \dots, T - (N - d)\}$ that refers to the next year. Since the misusing party is assumed to choose day d to minimize $DP(d)$, the achieved DP (DP^*) is given by

$$DP^* = \min_{d=0,\dots,N-1} DP(d), \quad (8)$$

and is used to decide whether an RI model achieves a required DP, i.e., $DP^* \geq 1 - \beta_{req}$.

For planners, achieved DP and SP are both useful for specifying input parameters for RI models: for a given RI model, specify a required DP and then check whether the resulting SP is satisfactory. If not, vary k and μ to adjust the resulting SP while still achieving the required DP. Or, specify a required SP (e.g., any facility should be inspected with at least 0.2 probability) and then check the resulting DP. If the resulting DP is not satisfactory, vary k and μ to adjust the resulting DP while maintaining the required SP.

3.3 Example Scenario

Consider $N = 365$ inspection days, $n = 3$ facilities in a State to which the RI scheme is applied, the signature time of $T = 90$ days, the misuse detection probability of $p_{id} = 0.9$, a minimum number of $k = 1$ RI, and the expected number of $\mu = 4$ RIs. Figure 2 plots the misuse DP for the three RI models of Section 2.2.

Figure 2 shows that RI model 1 gives the highest achieved DP among the three RI models for the above input parameters. The property of the DP curves of being constant up to a certain day d and then following a U-shape is due to the definition of RI models 1-3. Other RI models may lead to different shapes; see [3].

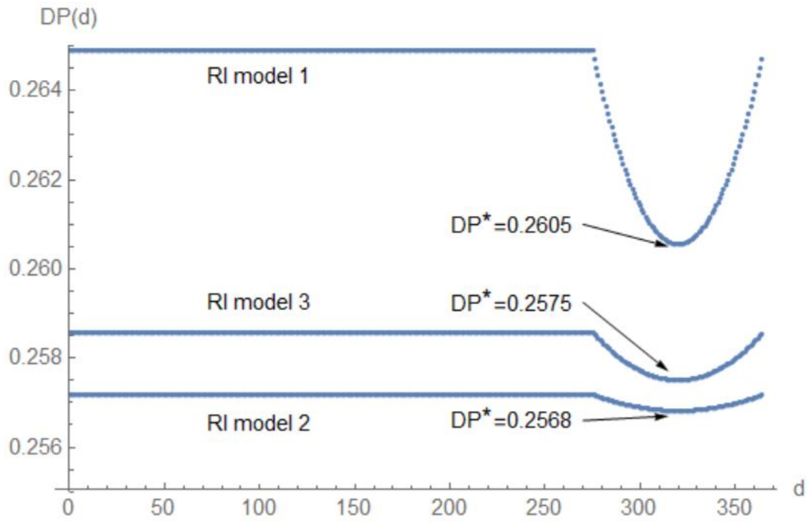

Figure 2: DP curves for the RI models 1-3 of Section 2.2.

Table 1 presents the achieved DPs for RI models 1-3 for various pairs (k, μ) and $N = 365$, $n = 3$, $T = 90$ and $p_{id} = 0.9$.

RI model 1

k↓ and μ ->	1	2	3	4	5	6	7	8	9
0	0.0726046	0.139987	0.20252	0.260549	0.314395	0.364357	0.410712	0.453718	0.493616
1		0.139987	0.20252	0.260549	0.314395	0.364357	0.410712	0.453718	0.493616
2			0.20252	0.260549	0.314395	0.364357	0.410712	0.453718	0.493616
3				0.260549	0.314395	0.364357	0.410712	0.453718	0.493616
4					0.314395	0.364357	0.410712	0.453718	0.493616
5						0.364357	0.410712	0.453718	0.493616
6							0.410712	0.453718	0.493616
7								0.453718	0.493616
8									0.493616

RI model 2

k↓ and μ ->	1	2	3	4	5	6	7	8	9
0	0.0713311	0.137627	0.199238	0.256494	0.309697	0.359133	0.405065	0.447739	0.487384
1		0.138588	0.199829	0.256819	0.309864	0.359213	0.405102	0.447755	0.487391
2			0.201103	0.257738	0.310454	0.359562	0.405294	0.447855	0.48744
3				0.259157	0.311567	0.360341	0.405797	0.448159	0.487613
4					0.313049	0.361569	0.406708	0.448784	0.488016
5						0.363068	0.407999	0.449782	0.488732
6							0.409485	0.451102	0.489784
7								0.452556	0.491107
8									0.492518

RI model 3

k↓ and μ ->	1	2	3	4	5	6	7	8	9
0	0.0713311	0.137627	0.199238	0.256494	0.309697	0.359133	0.405065	0.447739	0.487384
1		0.138805	0.200328	0.257501	0.310629	0.359995	0.405862	0.448476	0.488065
2			0.201422	0.258513	0.311565	0.36086	0.406662	0.449216	0.488749
3				0.259529	0.312504	0.361729	0.407465	0.449958	0.489436
4					0.313448	0.362601	0.408272	0.450704	0.490125
5						0.363477	0.409082	0.451453	0.490817
6							0.409895	0.452205	0.491513
7								0.45296	0.492211
8									0.492912

Table 1: Achieved DPs for the RI models 1-3 of Section 2.2, and for $N = 365$, $n = 3$, $T = 90$ and $p_{id} = 0.9$.

As for the SPs, there is strong numerical evidence in addition to the values in Table 1 that the achieved DP of RI model 1 is greater than the achieved DP of RI model 3 which is greater or equal to the achieved DP of RI model 2 for all pairs (k, μ) with $k < \mu$. For $k = 0$ the achieved DPs of RI models 2 and 3 coincide by definition of the RI models. The achieved DP for RI model 1 is constant for a given μ and any $k < \mu$, because Eq. (2) does not depend on k (see the comments at the end of the section on RI model 1).

3.4 RI Model selection

RI models should be compared based on multiple criteria. The most obvious criteria are to minimize the average annual number of RIs (μ) and maximize the achieved DP. However, widely varying annual numbers of RIs would be impractical for inspection planning and resourcing purposes. An upper limit on the annual number of RIs may be needed. Simultaneously, if the annual number of RIs does not vary enough then the misusing State gains some ability to predict whether an RI will occur close to the end of the reference period. The State could conceal an acquisition path by identifying and exploiting low-risk opportunities for facility misuse. These tradeoffs are complex and require further discussion. For the discussion here, it is assumed that a model is preferred that maximizes the achieved DP DP^* for given value μ (or minimizes μ while achieving a required achieved DP DP^*), regardless of other considerations.

Table 1 indicates that RI model 1 should be selected for implementation because for given pairs (k, μ) its achieved DP is always greater than the achieved DPs of RI models 2 and 3. The disadvantage of RI model 1 is that its number of RIs is $\lfloor \mu \rfloor$ or $\lfloor \mu \rfloor + 1$, i.e., highly predictable. Thus, if RI model 1 is applied year after year the misuser can observe that only $\lfloor \mu \rfloor$ or $\lfloor \mu \rfloor + 1$ RIs are performed, and thus, in a year in which $\lfloor \mu \rfloor + 1$ RIs have been performed, the misuser knows that there will be no further RI that year. Therefore, he directly starts the misuse if he did not do so before. Note that RI model 2 is – among the three considered here – the one with the highest variance of the number of RIs, i.e., the highest unpredictability in terms of numbers of RIs.

Could it also be that there is a fourth model from the set X (see Eq. (1)) that results in an even higher achieved DP compared to that of RI model 1 for all input parameters? Thus, we raise the question of RI model selection. If no further constraints are imposed (e.g., maximum unpredictability of the number of RIs in each year, a fixed upper value of the number of RIs, etc.), then an RI model from set X should be selected that maximizes the achieved detection probability, i.e.,

$$\max_{x \in X} DP_x^* \,, \tag{9}$$

where DP_x^* means the achieved DP according to Eq. (8) for a specific RI model $x \in X$.

Problem (9) is a tough one that has not been solved, but it bears resemblance to problems treated in the field of “calculus of variations”; see [6]. For example, consider a positive function $f(x)$ on the interval $[0, L]$ with $f(0) = a$ and $f(L) = b$ for given positive values a and b. Let $S(f)$ be the surface defined by rotating the curve around the x-axis. The area of the surface is then

$$S(f) = 2\pi \int_0^L f(x) \sqrt{1 + \left(f'(x)\right)^2} dx \,.$$

Consider the function space $V \coloneqq \{f: [0, L] \to \mathbb{R}: f \text{ differentiable on } (0, L), f(0) = a, f(L) = b, f(x) > 0 \text{ for all } x \in (0, L)\}$, then the variational problem

$$\min_{f \in V} S(f)$$

must be solved. In the context here, the function space is the set X of RI models given by Eq. (1) and the functional to be optimized is $\min_{d=0,\dots,N-1} DP_x(d)$. For curiosity, the solution of the above “surface” problem is the so-called catenoid; see [6].

4 Tool for Random Inspection Planning in Safeguards

The authors developed the Tool for Random Inspection Planning in Safeguards (TRIPS) to implement these statistical approaches in an evaluator-usable package. Figure 3 shows the input fields (left), main results (center), and the achieved DP as a function of the identification probability p_{id} and detection windows for various period lengths (right). TRIPS has aided in discussion of proposed safeguards practices and may serve as a prototype for future safeguards software.

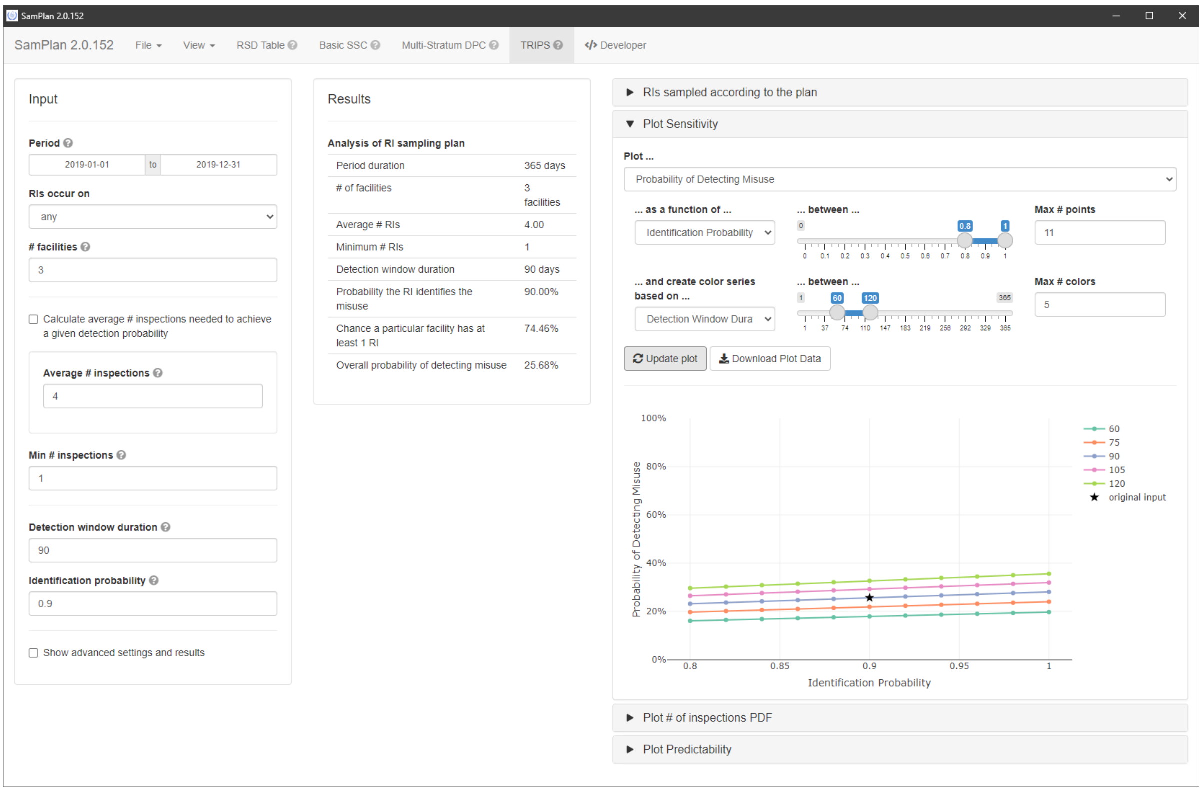

Figure 3. A screenshot of the TRIPS user interface, including input fields (left), main results (center), and auxiliary plots (right).

For a given misuse scenario (left input fields in Figure 3), TRIPS calculates the chance a particular facility has at least 1 RI during the year (SP), and the achieved DP (the two last entries in the middle column in Figure 3). The achieved DP can be calculated as a function of μ or vice versa. In the latter case, the box "Calculate average # inspections needed to achieve a given detection probability" must be ticked.

Various auxiliary plots can be created, including sensitivity curves that illustrate how μ, SP, and the achieved DP are affected by assumptions about p_{id} and T; see Figure 4 (left). Also, the probability mass function of the number of RIs can be illustrated; see Figure 4 (right).

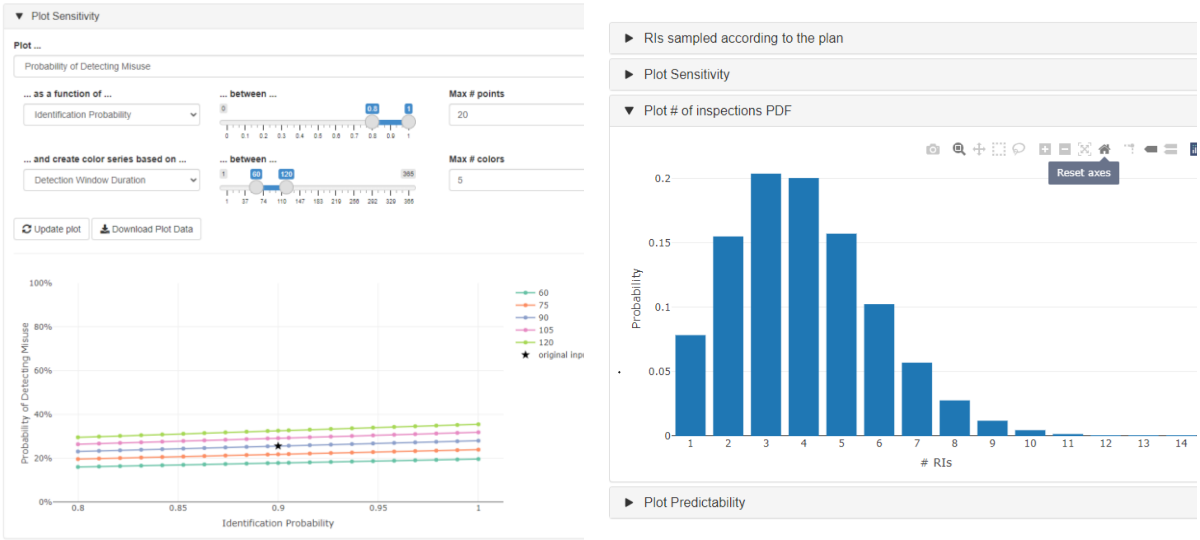

Figure 4. Sensitivity curves (left) and probability mass function of the number of RIs (right).

5 Conclusions and future work

This paper describes the basic framework for RI models and formalizes the objectives of RIs. As a result, the equations for the selection probability (i.e. the probability that a particular facility is selected at least once per year for an RI), and the detection probability (i.e. the probability of detecting the misuse within T days after its start), are given and discussed for three RI models.

A main future research topic consists in solving the optimization problem (9) using the set of RI models given by Eq. (1) with the possible extension $\mathbb{P}(\text{number of RIs} = k) > 0$, and extending it beyond the minimum-effort-maximum-achieved DP tradeoff discussed in Section 3.4. Practical considerations, such as minimizing inspection resource variability or maintaining a sufficient level of unpredictability, should also be considered. For this, additional statistical properties of the RI models need to be derived. Advanced RI models may need to be developed to achieve satisfactory performance in all considerations. Simulation studies may be needed to evaluate the RI models. Procedures for selecting and applying RI models remain a topic of discussion. Application and refinement of these models for planning and evaluation purposes of safeguards activities under the SLC remains a high priority for the IAEA.

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