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IMPLICATIONS OF LARGE TOTAL MEASUREMENT UNCERTAINTY IN POLICING THE CRITICALITY SAFETY LIMIT

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ABSTRACT

The fissile gram equivalent (FGE) mass assigned to an item via nondestructive assay (NDA) and its corresponding total measurement uncertainty (TMU) estimate are often used to determine if the item's contents fall below an administrative criticality mass limit for that class of items. Overly simplistic models of the TMU, incorrect application of these uncertainties, or a combination of both can result in improper dispositioning of items with actual contents that exceed both the appropriate administrative and criticality safety mass limits. This incorrect assessment results from the erroneous assumption that the assay result, from which the uncertainty is derived and assigned to, is the same as the true content of the container rather than only an experimental estimate. The traditional uncertainty analysis applied to NDA provides an uncertainty that represents the range of assay results consistent with the true mass value, whereas the uncertainty needed should represent the range of true mass values that are consistent with the assay result. For small relative TMU, the impact is usually insignificant because as the assay value and true (but unknown) values should be "close," but as uncertainties become larger (e.g., greater than 20%), the potential to mischaracterize an item relative to the criticality safety limits becomes non-negligible. This paper examines the potential impact of large TMU on the proper dispositioning of such items.

INTRODUCTION

In non-destructive assay of contact handled (CH) waste items, estimated measurement uncertainties in excess of 30% (at the 1 σ level) [1], [2], [3], [4] are not uncommon, and measurements of intermediate and remote handled (RH) wastes may have associated uncertainties assignments which are 100% or more. Nonetheless, these high uncertainty measurement results are used in the determination of the disposition path of these items. For example, typical criteria used in policing the criticality safety limit using NDA has been that ($m + 2 \times \sigma_{TMU}$) must be less than a safe mass limit defined for the item type [5], where m is the assay mass value of nuclear material and σ_{TMU} is the TMU. If this condition holds, then the item mass is assumed to have no more than a 2.5% chance of exceeding the safe mass limit and so meets the criticality disposition criteria. However, this approach has several fundamental flaws including

- the assumption that the uncertainty in the reported assay result is symmetric;
- the even stronger assertion that the uncertainty in the reported assay result may be represented by a normal (Gaussian) distribution; and
- the assumption that the reported assay result is equal to the true mass value of the item.

If we consider an assay system with fixed relative total measurement uncertainty of ±100%, it is assumed that a mass assay result of 1 g with an uncertainty of ±1 g (±100%) at the 1 σ level, implies that the actual mass is less than 3 g (m_{assay} + 2 × TMU), and based on this assay result, the contents would be assumed to be well below the typical criticality safety limit of 200 g. However, without considering the measurement response from higher mass loadings, such an assumption cannot be justified. With the 100% uncertainty, the assay of any mass loading has a non-negligible chance to report at 1 g or less. For instance, a true 1 kg mass would have an even greater chance of reporting below 1 g if the relative TMU is 100%, and it is therefore clearly wrong to report 1 g ±100% in such cases because the true mass is 1000 g! Therefore, to police the criticality safety limit, the TMU analysis must define the potential mass loadings consistent with the assay result. This is true even if the measurement system is only intended for gram level mass loadings.

Note, although we are concerned primarily with whether items are below safe limits, when the relative TMU becomes large the breakdown of the historical interpretation is also evident in the other direction. A reported value of (100 ± 50) g would suggest a reasonable chance that the item contains a negative amount of material. Since it is not possible for there to be less than zero mass present, we see that the use of the normal distribution is inappropriate.

The reported TMU is almost always dependent on the true mass value of the item. In practice, because the true mass value is unknown, the uncertainty in the NDA result is determined based on the assay mass value. The difference between the assay result and the true mass value (the measurement error) also introduces an error in the calculated TMU which increases as the measurement error increases. The typical uncertainty analysis describes the distribution of possible assay results about the true mass value; however, as the TMU becomes large, it becomes necessary to ask "what is the range of true mass values that could result in the reported assay mass?"

IMPACT OF LARGE TMU ON THE FGE

To police the criticality limits for a package, the FGE mass is determined via NDA, and a TMU is determined for the assay result. For example, the Transuranic Waste Acceptance Criteria for the Waste Isolation Pilot Plant (DOE/WIPP-02-3122) requires the measured FGE mass plus twice the TMU to be compared to the administrative criticality mass limit for each package, and the result must be less than the cutoff value. In other words, the assay result must be less than the FGE limit for the container divided by a factor of (1 + 2 × the fractional TMU) to meet the administrative control requirement. Applying this method to the measurement results, if the TMU is 20%, then the effective FGE limit for a 55-gal drum is 142 g. Similarly, the effective FGE limit would be 100 g for 50% TMU or 66 g for 100% TMU. Unfortunately, while reported TMU values of 50% or greater may be sufficient to meet the requirements of the waste acceptance criteria, the assay results are meaningless with respect to criticality safety. This is because once the TMU approaches 50% at the 1 σ level, all assay results are consistent with any true mass value at the 2-σ confidence level. In other words, if one takes the TMU at face value and accepts that it describes the assay value in a statistical sense, then the quality of the measurement is very poor and the mass value is not well located.

The error contributions to the TMU analysis for waste package assay are generally treated following the central limit theorem [6], meaning the variance is estimated by summing the variances of the individual contributions and the final probability density function is taken as approximately normal. However, for the large uncertainties encountered in waste assay, the approach is too simplistic. Both random and systematic error contributors are treated as Gaussian in shape and random in nature, although this is often a coarse approximation (Figure 1) and measurement sensitivity can vary by orders of magnitude across the extent of

the item (Figure 2). The individual uncertainty components are added in quadrature even if the error component is asymmetric or single sided. Specifically, the TMU at 1-σ is estimated by quadrature summation is the square root of the sum of variances of each source of uncertainty. This generates a formal measure of the "width" of the distribution but says nothing about the shape of the distribution and its tails. In addition, the TMUs are effectively calculated assuming that the assay mass is equal to the true mass within the container when in fact there could be a substantial difference between the two. The simplistic approach is generally acceptable if the TMU is relatively small (e.g., <20%) and when a known item (i.e., a standard mass) is assayed with the aim of quantifying the assay recovery of the known items. However, as the uncertainties increase, this simplistic treatment leads to increasingly "incorrect" TMU assessments for the unknown FGE mass and potential violation of the administrative criticality control limits.

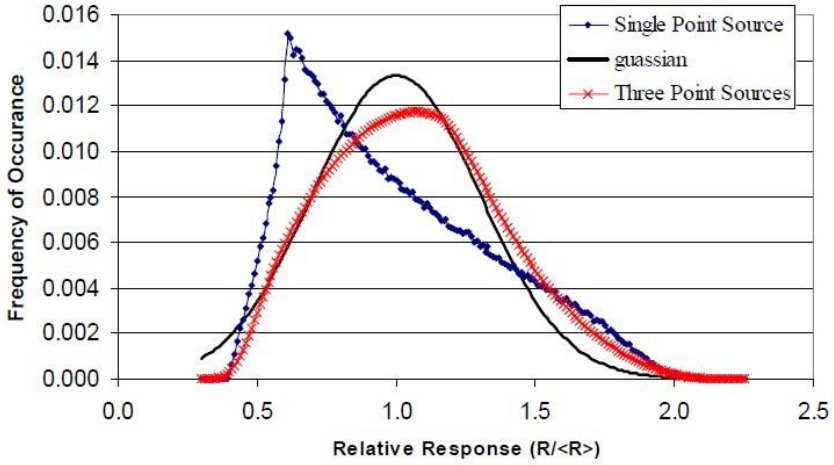

Figure 1. Example of an asymmetric error contributor [7] **typically represented as a normal distribution: Simulated response distribution from a 200 L drum gamma-ray assay system for a single randomly positioned point source and three sources randomly positioned within the drum. As the number of point sources increases, the point source distribution error approaches a normal distribution.**

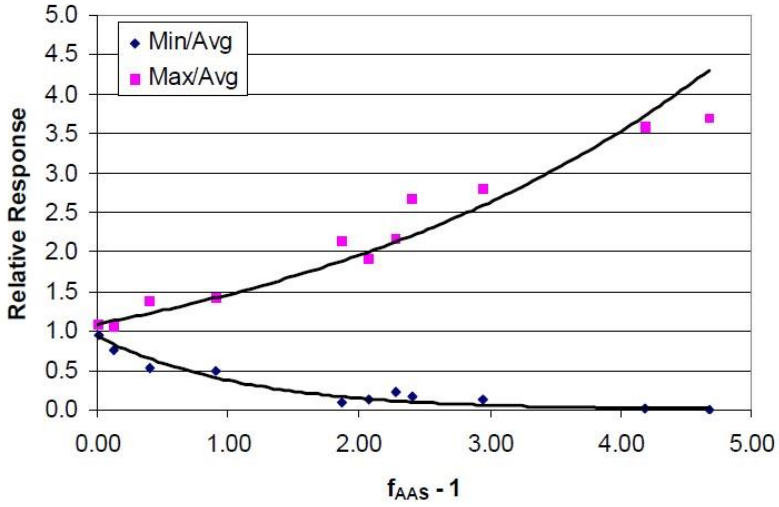

Figure 2. Extremes in response for a passive neutron coincidence drum counter as a function of increasing moderator content [7]**. As the moderator content increases, the relative response from the interior portions of the drum are significantly reduced, creating effective blind spots in the assay.**

Considering gamma attenuation leads to an appreciation of the breakdown of the normal distribution. The attenuation correction factor has an exponential form, $f = e^t$, where t is the optical thickness of the attenuating material. In calculating such corrections when t is small and well estimated, we can linearize the problem and find $f \approx (1 + t) \pm \sigma_t$, and the distribution of f follows that of t. But when t is large and poorly determined, the variation of f about the expected value is large and best described in multiplicative terms. That is, rather than being 3% higher or 3% lower, the result might be a factor of three higher or a factor of three lower (but still positive). And to linearize, we use $ln(f) = t$, which leads to the log-normal distribution rather than the normal distribution.

The shortcomings of the large TMU will be illustrated by considering the assay of two items—one containing 120 g FGE and the second containing 500 g FGE, first with a 1-σ TMU of 10% and then with a TMU of 50%. For this exercise, we will assume that the material is contained within a 200-L drum, and the fissile mass limit is 200 g. The administrative limit is defined as the FGE_{limit} less 2 × TMU(FGE_{limit}) for each item type, and the measurement result, m_{assay}, must be below this value to be suitable for disposition. Alternatively, the administrative limit can be set relative to the measurement result using the relation, m_{assay} + 2·TMU(m_{assay}) < FGE_{limit}. For small TMU values these limits are equivalent.

1) If the TMU for a measurement is ±10%, the deviation between true, and assay mass will be small enough that the reported mass and TMU are reasonably close to the true mass and associated TMU value. Using the first method to ensure that the contents of the container are below the 200 g FGE fissile mass limit, we would set an administrative limit of 160 g FGE (i.e., 200 − 2 × 0.10 × 200). Assuming the uncertainty follows a normal distribution, the assay result for the item containing 120 g FGE will be less than 144 g 97.7% of the time, will have nearly zero probability of exceeding the administrative limit, and will be accepted for continued dispositioning. The container with 500 g FGE would be expected to yield an assay result less than 400 g FGE (500 − 2 × 0.10 × 500) less than 2.5% of the time and less than 200 g FGE with a probability of 1 in 10^9 (6 σ) and will be reliably rejected.

2) However, if the TMU for a measurement is ±50%, the assay becomes effectively meaningless for criticality control purposes. In this case, the manner of that the administrative limit is defined is important. If the FGE_{limit} is policed by following the expression m_{assay} + 2 × TMU(m_{assay}) < FGE_{limit}, the effective administrative limit would be 100 g FGE. However, if the administrative limit is set using FGE_{limit} – 2 × TMU(FGE_{limit}), then the administrative limit is 0 g FGE. This is because in this example the 2-σ uncertainty always encompasses 0 g as a potential assay result. The possible assay results for the (nominal) 120 g item span the range from 0 to 240 g, and the (nominal) 500 g item spans the mass range from 0 to 1000 g. The probability distributions of potential assay results for TMUs of 10% and 50% are illustrated in Figure 3. As illustrated in the TMU = 50% plot, if the statistical interpretation of the 50% TMU is taken literally, it is not possible to show that a 0 g FGE assay result does not exceed the administrative limit (or any limit for that matter).

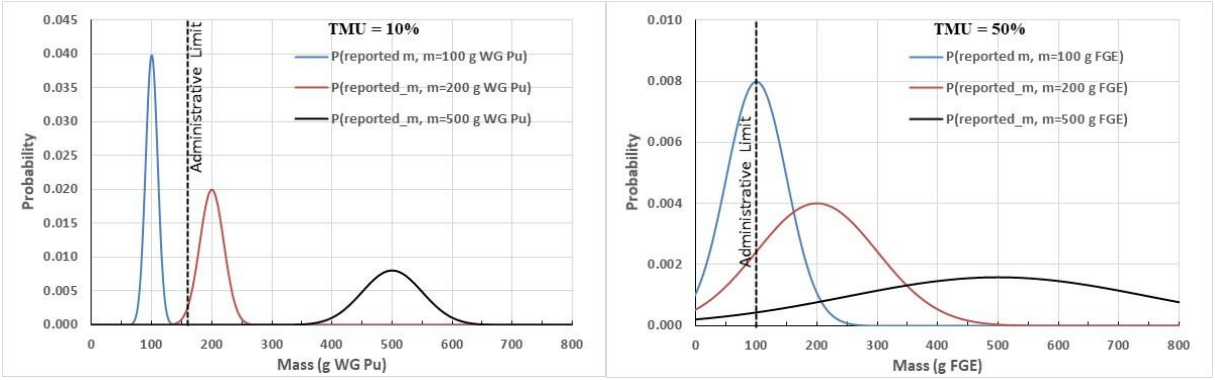

Figure 3. Probability distributions for the mass assay result for two different true mass loadings (100 g and 500 g FGE), assuming a 1-σ TMU of ±20% (*left*) and ±50% (*right*) and based on a normal distribution.

The plots illustrate that if the TMU is ±50%, the assay of the 500 g FGE item has a non-negligible probability of yielding a mass result of less than 100 g FGE (16% of the time). Therefore suppose, for the purpose of illustration, that the assay value obtained for such a drum is 100 g FGE. Because the TMU analysis for the systems examined assumes that the assay result is equal to the true mass, the value of FGE + 2 × TMU for this assay is therefore 50 + 2 × 0.50 × 50 = 100 g FGE. It would be concluded incorrectly that the fissile contents of the 500 g item were less than the FGE limit. Note that for a given waste package, replicate measurements will likely not alter the outcome because the large contributors to the TMU may be random on a population basis but not for a specific item (e.g., point source effects) introducing strong item-specific biases. The randomness assumed in the current interpretation is actually more restrictive in nature and applies to the population of packages. On average, we may achieve the expected level of assay performance, but on the particular item there will be dramatic variation. Note again that broad Gaussian distributions admit negative assay values, which is clearly nonphysical and another indication that when the fractional TMU is large, a different way of setting tolerance limits is needed. Specifically, a method that admits only positive outcomes and takes into account the non-Gaussian shape of the uncertainty distribution is necessary.

Alternatively, we can examine the probability that the true mass will result in an assay mass result below the administrative cutoff limit. Consider the measurement of an item where the criticality safety limit is 200 g and the TMU for the waste stream is described as a normal distribution and the 1-σ TMU value is declared as 60%. The typical approach to policing the criticality safety limit is to set an administrative limit where the assay mass plus twice the TMU value is less than the criticality safety limit. In this case, the administrative limit would be 200/(1 + 2 × 0.6) = 90.9 g. The probability as a function of true mass that the reported mass will be less than the administrative limit of 90.9 g is shown in Figure 4. The plot illustrates that true masses of 200 to 400 g have a 10% to 17% chance of reporting below the administrative limit or five times the goal rate.

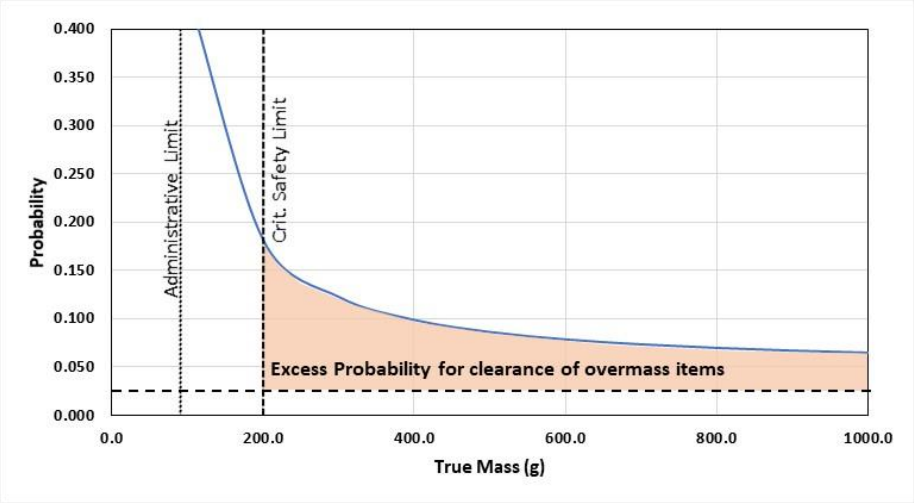

Figure 4. Probability as a function of true mass of the assay result being reported below the administrative cutoff limit where the TMU is described by a normal probability distribution with a TMU of 60% (1 σ). For this example, the measurement would not be adequate to show that any item with true mass above the safety limit would yield an assay result above the administrative limit with greater than 95% confidence.

There is generally an implicit assumption that the administrative limit is overly conservative because the assay result for a true mass of 90.9 g has less than 2.5% probability of exceeding 200 g. However, because the uncertainty is described as a normal or Gaussian distribution, masses greater than the criticality limit have a greater than 5% chance of assaying below the administrative limit. Figure 5 provides the required administrative mass limit for the assay result to ensure that the item contains less than the criticality safety limit. As seen in the plot, once the uncertainty exceeds 50% at the 1-σ level, the administrative limit becomes negative, and no assay result provides the required confidence needed to safely disposition the item.

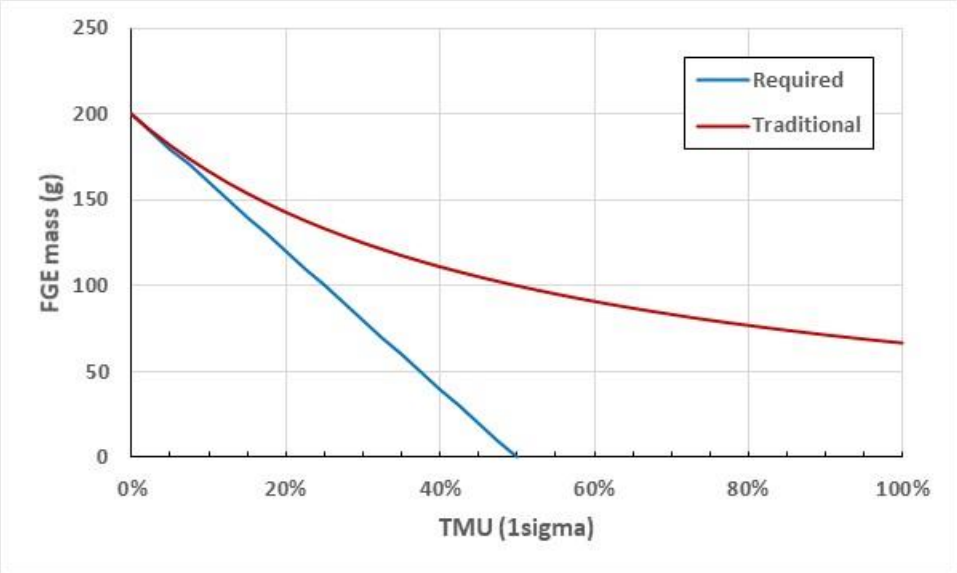

Figure 5. Administrative cutoff limit for the reported mass as a function of TMU required to ensure that the item content has less than a 2.5% chance of exceeding the criticality safety limit. The traditional limit is shown for comparison.

The determination of an administrative cutoff limit is complicated by asymmetric uncertainty contributors. Point source effects as illustrated in Figure 2 can not only broaden the uncertainty distribution but bias the distribution relative to that expected for a normally distributed error. Figure 6 provides a comparison of the 2σ mass limits as a function of the assay mass result determined using a full TMU analysis of a reasonably moderating combustibles drum in a passive neutron drum coincidence counter. Summation of the uncertainty contributors in quadrature would provide a TMU of approximately 20% (1σ) but the point source effects result in a greater tendency to underreport and the need to lower the administrative limit for dispositioning.

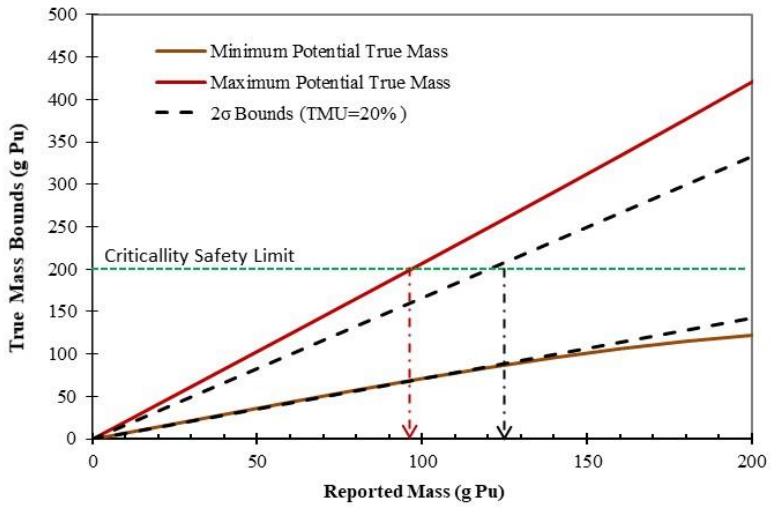

Figure 6. Plot of the bounding minimum and maximum true mass values as a function of reported mass for a passive drum coincidence counting system assay of a heavy combustibles drum. The minimum and maximum potential true mass curves represent the 2σ mass limits as function of the assay result based on the full TMU analysis. For comparison the 2σ bounds for a fixed 30% TMU are also shown. The required administrative cutoff limits to ensure 200 g FGE is not exceeded are indicated by the two arrows.

An approach often used to dismiss discussion of the impact of the large TMU is to invoke process knowledge. That is, historical data or other information is examined to place an upper bound on the possible mass contained within an item. This information could be as simple as the limit on the total material processed or a physical limitation of the container (e.g., insufficient volume to allow the mass limit to be reached). Alternatively, the use supplemental techniques (such as radiography) may be used to place an upper bound on the contents of an item. These techniques have historical precedence and are well founded. However, in these cases, if the upper mass limit on an item's contents is bounded by process knowledge, the mass limit is not being policed by the measurement, and its TMU is no longer relevant (Figure 7). If the additional information is being used to support the measurement and its associated TMU value, the information needs to be fully documented and incorporated into the TMU analysis. Heavy reliance on expert judgement needs to be justified, and records of decision maintained. If the documentation supports an upper bound on the fissile mass contents of the containers, applying such a limit implies that the assumption of a normal distribution for the uncertainties is incorrect and an alternative distribution must be established.

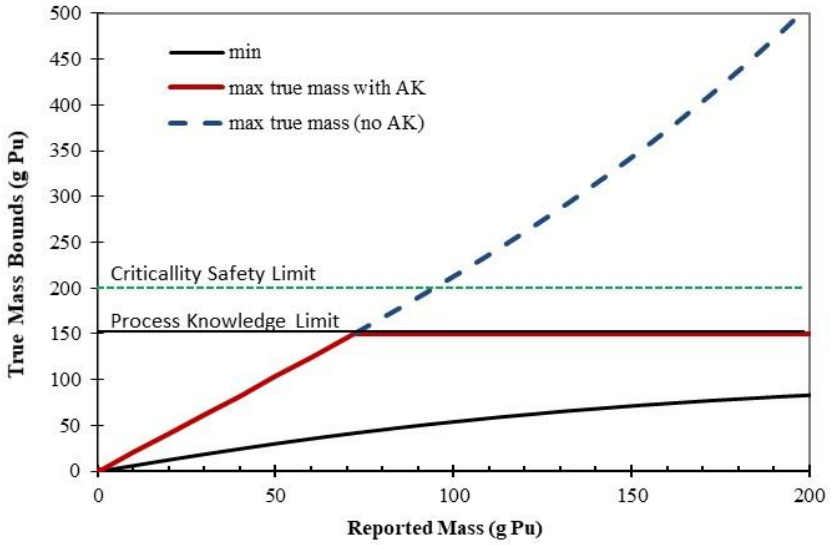

Figure 7. Impact of the imposition of an upper bound for the true mass values as a function of the reported assay mass. Imposition of an upper mass limit redefines the TMU assessment.

The criticality safety analysis (for a facility or a repository) is not based on the results of a single measurement but the aggregate of measurement results for the population of items. Usually at each step a conservative view is taken. The measurement results are used to ensure that fissile mass of local populations with a storage array and the overall population within a facility do not exceed levels where a criticality event might occur. An upper mass value is defined for each storage array or facility, and although the individual measurement results could be summed with their corresponding TMU values and disposition process halted once the safety limit has been reached, the more commonly used approach is to define a mass assay limit for each item based on the total fissile mass limit for the array divided by the maximum number of possible positions within the array. It is accepted that a fraction of the assay results will exceed the fissile gram limit for each item. The acceptable fraction, typically 2.3%, is based on an assumption about the probability distribution of true masses relative to the assay mass result. If the TMU analysis does not accurately reflect the population of items, the safety analysis is invalidated.

For measurements applied to criticality safety, the probability of underreporting the mass value is of more concern than overreporting it. The uncertainty contributors consist of both systematic and random error sources. Certain uncertainty contributors may result in either random or systematic uncertainties depending upon the implementation of an appropriate bias correction (e.g., the multiplication effect in neutron coincidence assay produces a positive bias unless a typical case correction factor is applied, then the uncertainty would be considered to be random). However, for expediency, all uncertainties are often treated as random in nature. By proper treatment of the biases, they may be treatable as one sided (conservative or optimistic) errors potentially providing a more realistic uncertainty analysis allowing a more beneficial administrative limit to be applied.

Although this discussion has focused on NDA CH waste assay applications, the arguments apply to all large TMU measurements applied to policing the criticality boundary. In-situ mass determinations (e.g., hold-up measurements) and remote handled items where direct measurement of the FGE mass is extremely challenging require, particularly detailed and defensible uncertainty evaluations. Because information on the materials and geometry are difficult to obtain use of “conservative” assumptions is commonplace. However,

it must be cautioned that these assumptions must be well documented and defined, and it must be demonstrated in what way they are conservative. The impact of these assumptions on the assay mass result and TMU must be justified. Finally, because the TMU associated with these measurements is typically greater than 50%, the asymmetries in the uncertainty components must be taken into account in order to properly apply the measurement results.

CONCLUSIONS

The key question to be addressed in policing the criticality safety limit by nondestructive assay or other measurement techniques is "what is the probability that a true mass exceeding the criticality safety limit resulted in the reported assay value?" We have shown that measurement systems yielding total uncertainties of 50% or more are not suitable for policing the criticality safety limit without the integration of additional information because they are not capable of answering that question.

Reliance on crude (approximate normal distributions) high TMU measurement results for policing the criticality safety limits is not practical. At its simplest this is because once the TMU exceeds 50%, all true mass loadings have a non-negligible chance of reporting not only below the administrative mass limit but also reporting below the measurement's minimum detection level. For additional context recall that at the detection limit (probability of false positive equal to probability of false negative equal to 5%) the statistical uncertainty is about 30%; so 50% is a large uncertainty, although not uncommon for difficult-to-measure items.

Simply stating the TMU as a fixed percentage of the assay result is convenient and provides a useful comparison of the relative performance of one measurement system to another. However, these values do not adequately describe the system performance relative to the fissile mass limit for an item. For example, biases in a gamma-ray measurement tends to underreport at high mass values, whereas biases in a passive neutron system tend to overreport relative to the true mass value. Source distribution error contributions are largely asymmetric unless the fissile mass is distributed across multiple point sources. These error contributors tend to dominate the TMU for mass loadings near or beyond the FGE limit. To accurately predict the probability that a given item exceeds the fissile gram limit, a more sophisticated treatment of the uncertainties is required. One approach, for example, would be to us Monte Carlo sampling of the distributions describing the various random and systematic uncertainty contributions to generate plausible confidence bounds.

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